Interactions of Knowledge and Strategies

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Motivation and Background

- **Perfect Information**: fixed-point characterisations of ATL operators

$$\langle\langle\Sigma\rangle\rangle G\phi \quad \leftrightarrow \quad \phi \wedge \langle\langle\Sigma\rangle\rangle X\langle\langle\Sigma\rangle\rangle G\phi \tag{1}$$

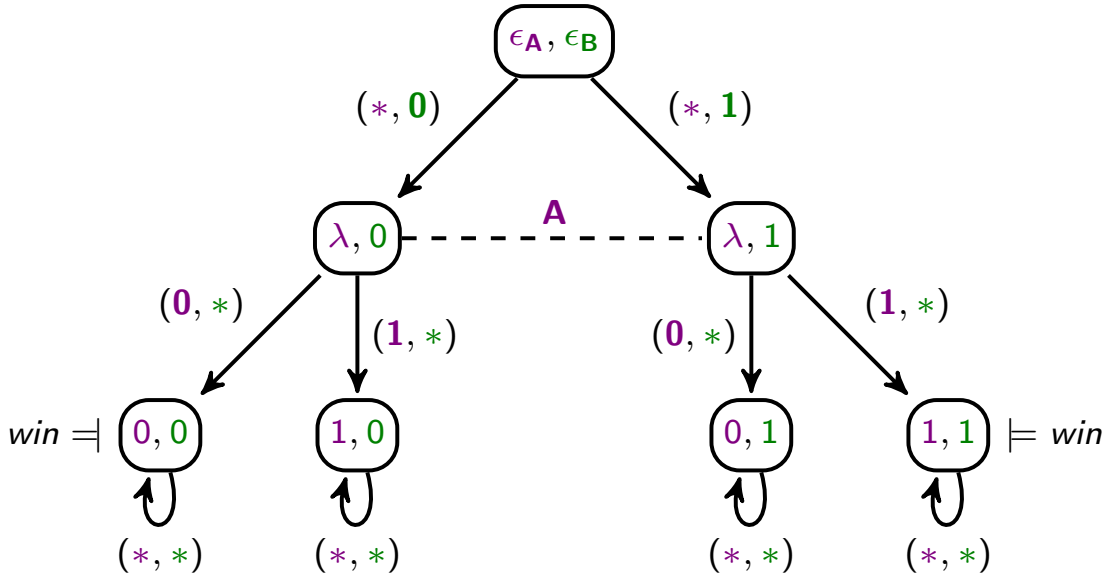
$$\langle\langle\Sigma\rangle\rangle F\phi \quad \leftrightarrow \quad \phi \vee \langle\langle\Sigma\rangle\rangle X\langle\langle\Sigma\rangle\rangle F\phi \tag{2}$$

$$\langle\langle\Sigma\rangle\rangle(\phi U\phi') \quad \leftrightarrow \quad \phi' \vee (\phi \wedge \langle\langle\Sigma\rangle\rangle X\langle\langle\Sigma\rangle\rangle(\psi U\phi')) \tag{3}$$

- **Useful Validities**: techniques for satisfiability [GS09] and model checking [AHK02, BDJ10]
- **The Problem**: (1)-(3) do not hold in the imperfect information semantics!

The Problem

ATL with Imperfect Information



- **B**ob chooses secretly between 0 and 1
- at the next step **A**nne also chooses between 0 and 1
- **A**nne wins the game iff the values provided by the two players coincide
- the dotted line indicates epistemic indistinguishability

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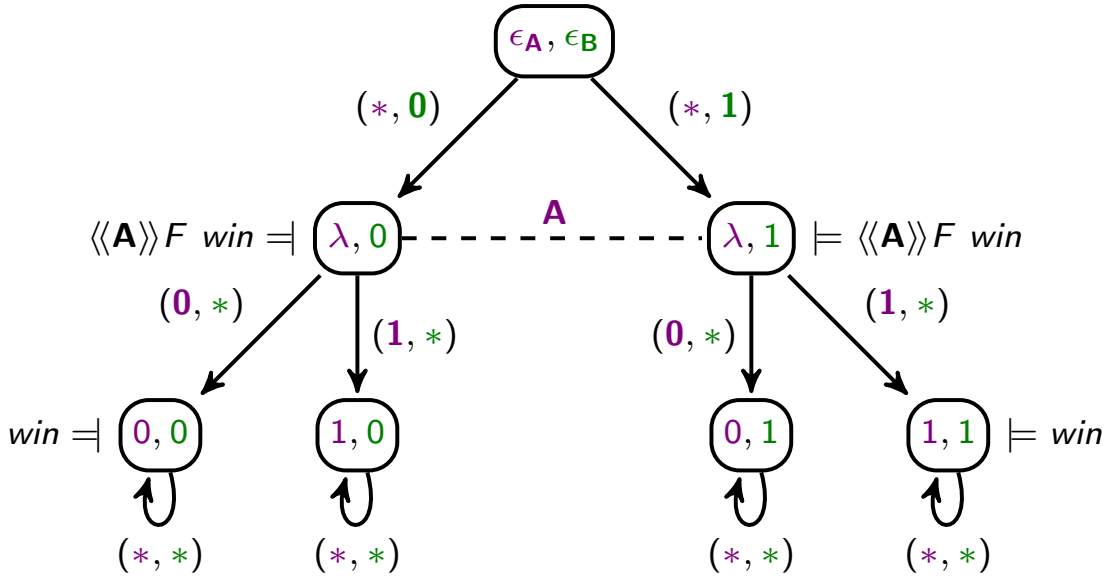

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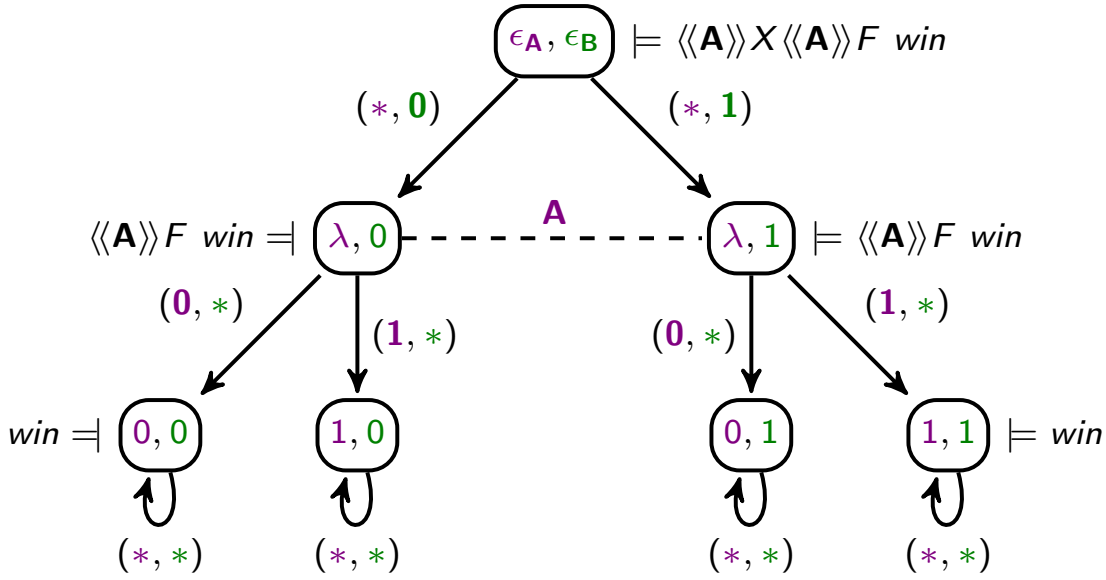

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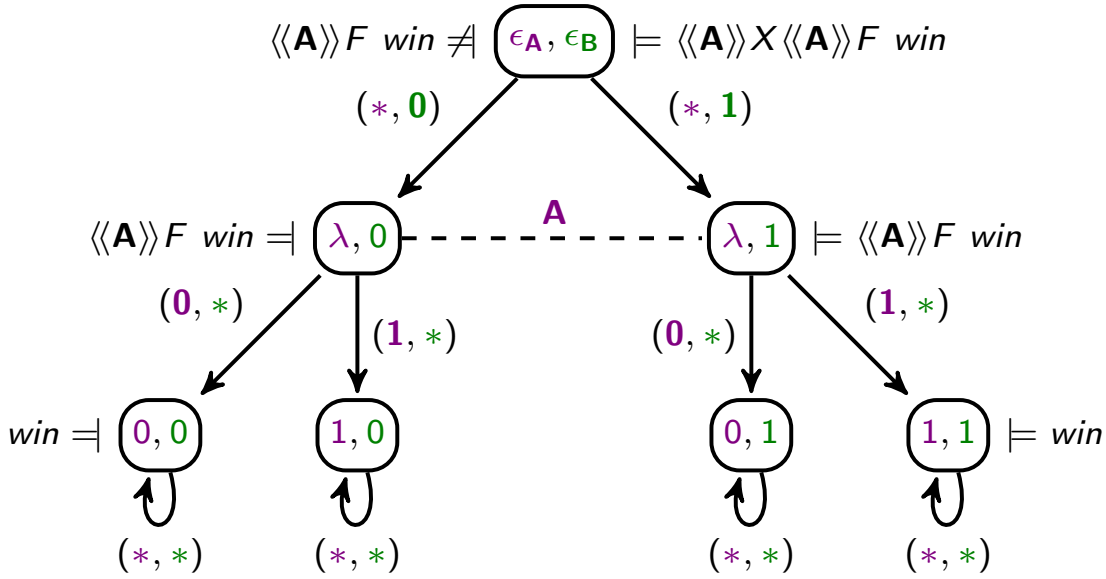

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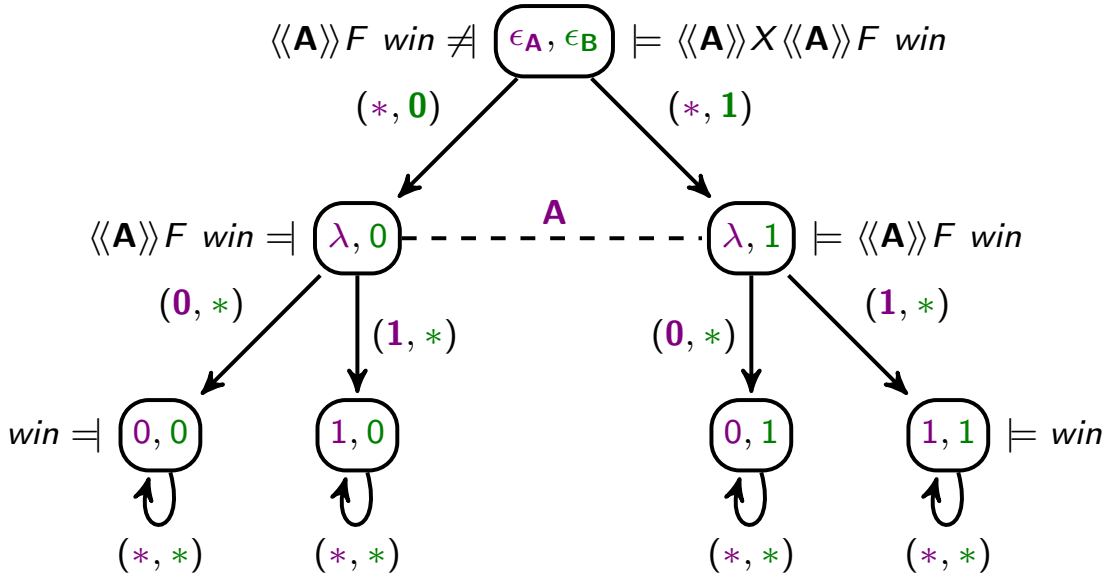

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- at the next step **A**nne also chooses between 0 and 1
- **A**nne wins the game iff the values provided by the two players coincide
- the dotted line indicates epistemic indistinguishability
- **A**nne knows that there exists a strategy to win the game . . .

  . . . however, she is not able to point this strategy out

  $\Leftarrow$ **A**nne has *imperfect information* of the game

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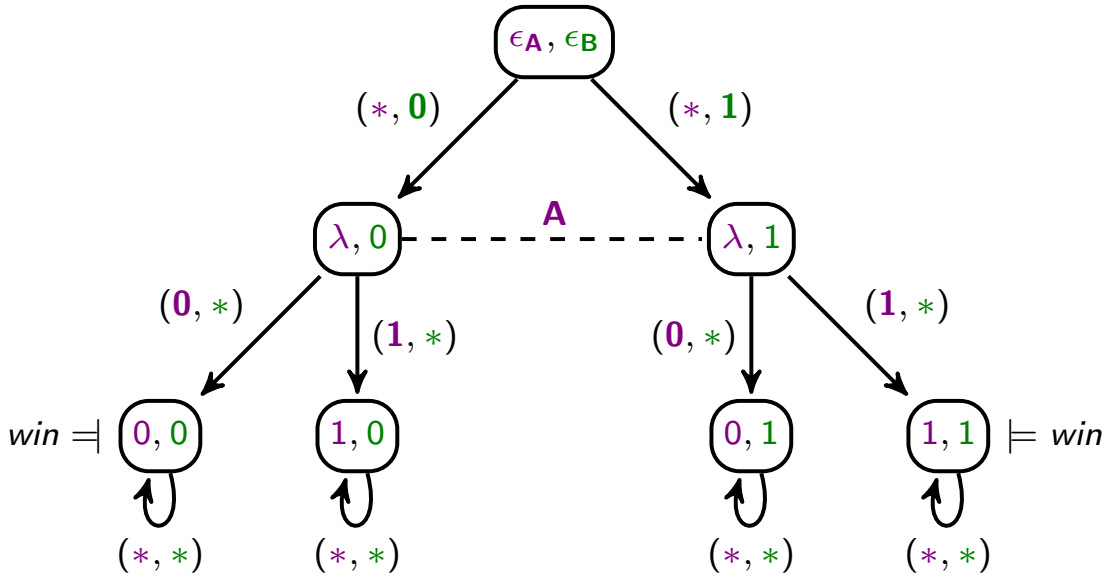

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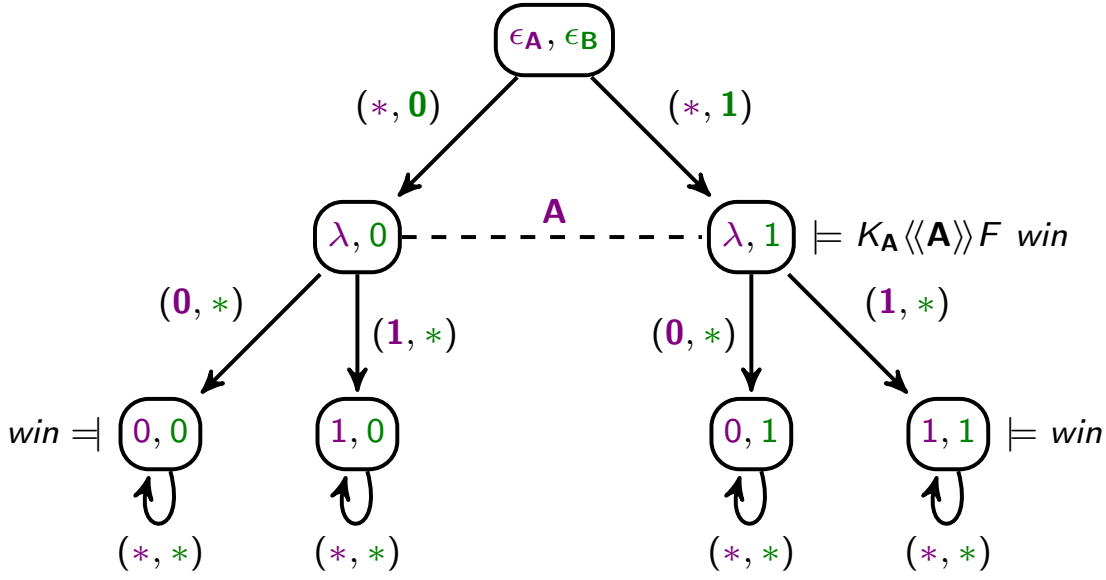

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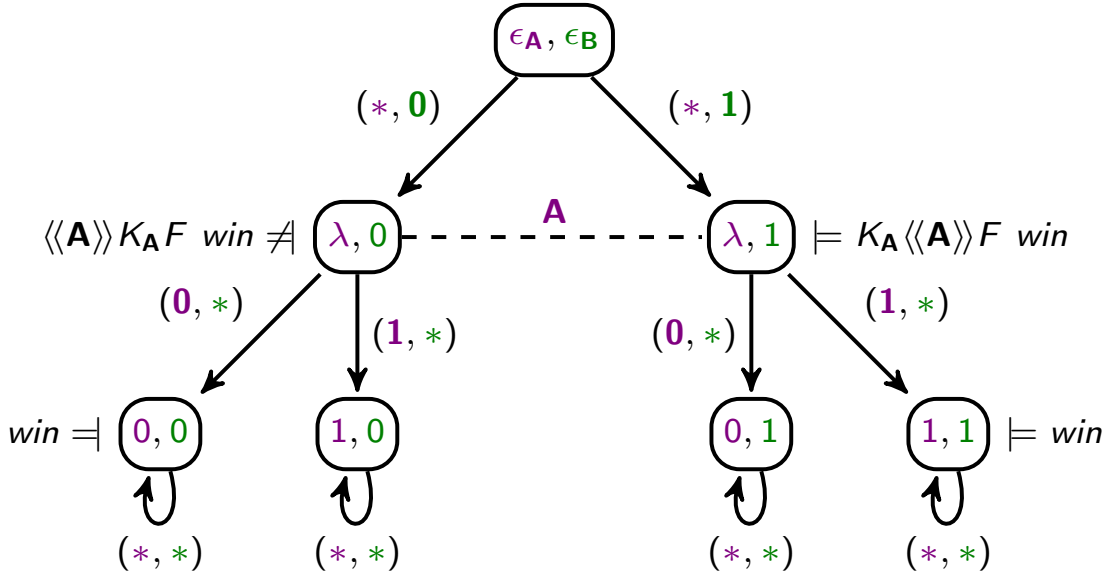

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- **A**nne knows that there is some strategy to win (knowledge *de dicto*)
- but there is no strategy known to her to guarantee a win (knowledge *de re*)

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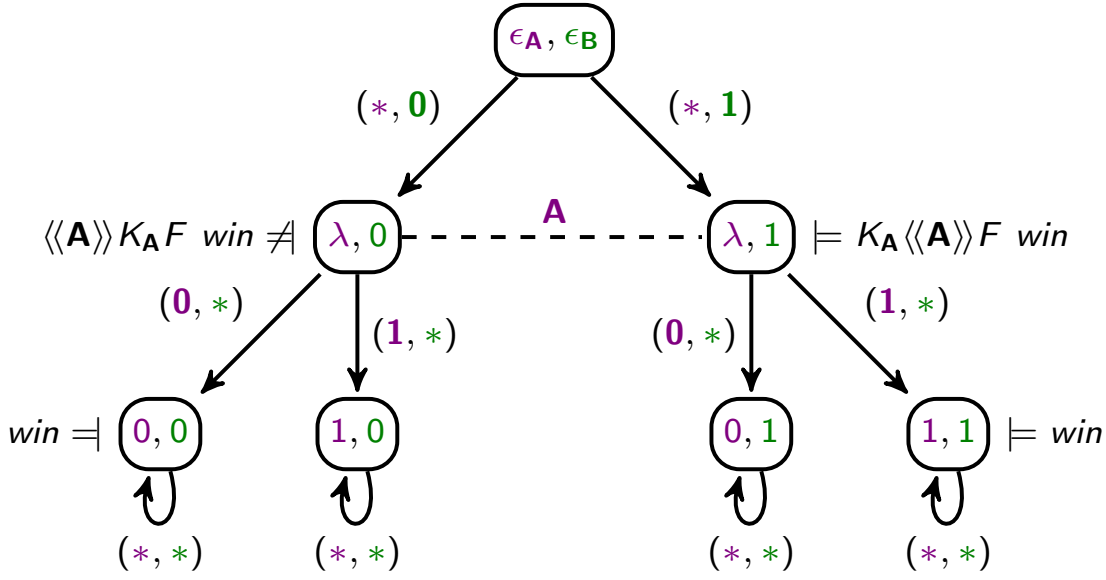

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Is there any way of combining ATL and epistemic operators so as to obtain something similar to (1)-(3)?

References

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