Automated Verification of Multi-Agent Systems
Why, What, and Especially: How?

Catalin Dima and Wojtek Jamroga

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Thank You's

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- Thomas Agotnes, University of Bergen
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- Juergen Dix, Clausthal University of Technology
- Valentin Goranko, Technical University of Denmark
- Wojciech Penczek, Polish Academy of Sciences

All the mistakes are, of course, ours.

Basic Reading

- Yoav Shoham and Kevin Leyton-Brown (2009), Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations. Cambridge University Press.
- Edmund M. Clarke, Orna Grumberg and Doron A. Peled (1999), Model Checking. MIT Press.
- Christel Baier, Joost-Pieter Katoen (2008), Principles of model checking. MIT Press.
- Wojciech Jamroga (2015), Logical Methods for Specification and Verification of Multi-Agent Systems. Available for free at: `http://krak.ipipan.waw.pl/~wjamroga/papers/jamroga15specifmas.pdf`

Outline

Part 1: Introduction to Model Checking for MAS

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1.1 Motivation

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Formal specification and **verification** helps to:

- Clarify and disambiguate the requirements
- Uncover the implicit assumptions about the system and participants

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- Uncover the implicit assumptions about the system and participants
- Sometimes, we can even prove that the requirements hold wrt the assumptions
- ...or disprove and generate a counterexample

Formal Verification of Strategic Ability

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- Functionality $\approx$ ability of authorized users to complete some tasks
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- One can try to formalize such properties in modal logics of strategic ability, such as ATL or Strategy Logic
- ...and verify them by model checking

Motivating Example: Security of Voting

Voting Scenario

Citizens of Pneumonia are voting in the presidential election.

There are n voters, each of them supposed to enter a voting booth at a polling station, select one of the candidates from the ballot, register their vote, and exit the polling station.

There is also a coercer who can attempt to bribe or blackmail the voters into voting for a particular candidate. The coercer can interact with the voters, e.g., making demands or giving instructions on how to vote. He is also capable of intercepting unencrypted messages.

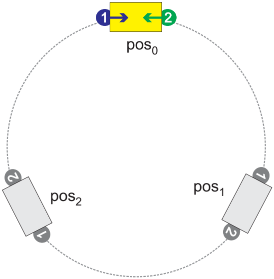
Part 1: Introduction to Model Checking for MAS

1.2 Modeling MAS

Modeling Multi-Agent Systems

- How to model a distributed system? $\rightsquigarrow$ **transition graph**
- Nodes represent states of the system (or situations)
- Arrows correspond to changes of state

Example: Robots and Carriage



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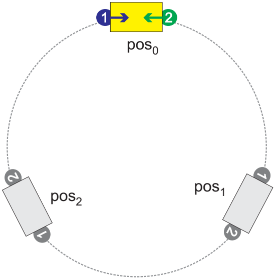

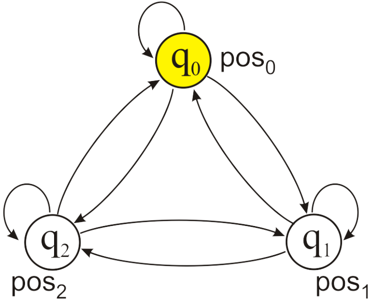

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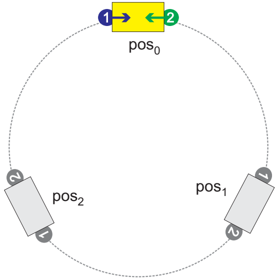

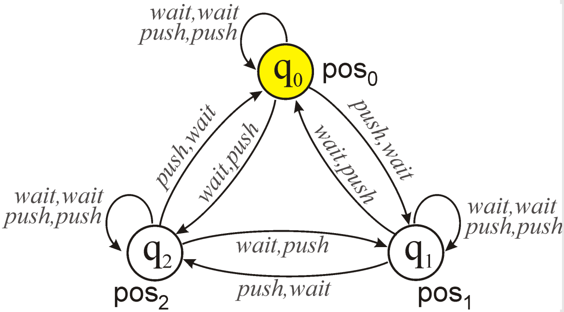

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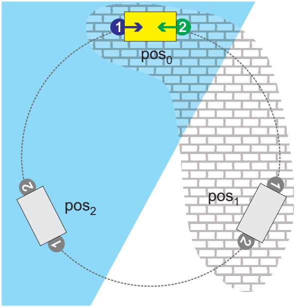

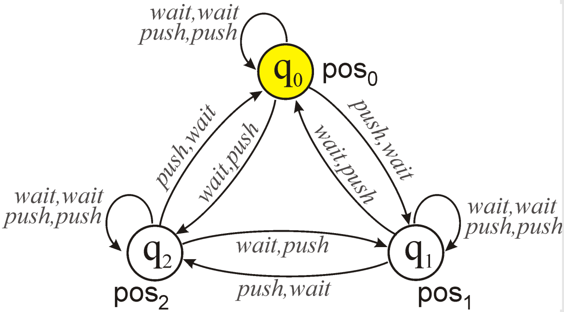

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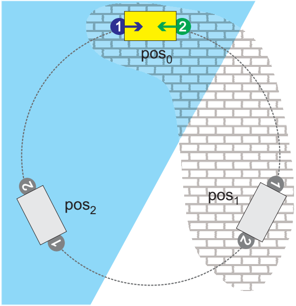

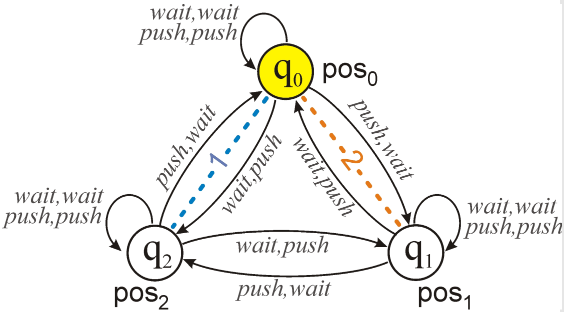

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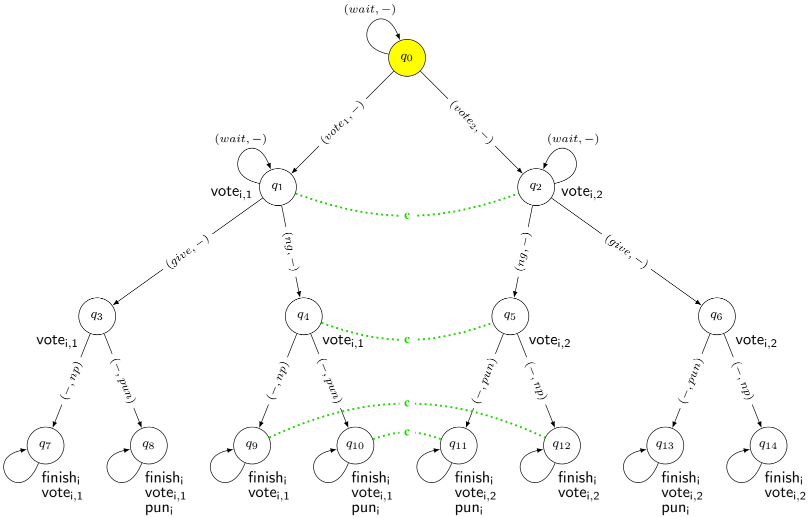

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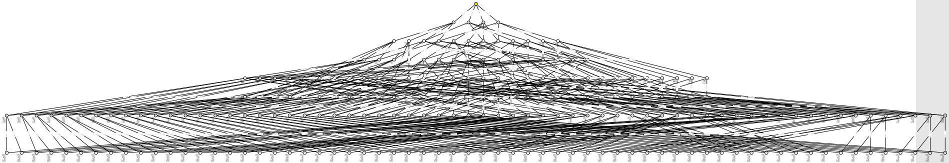

Part 1: Introduction to Model Checking for MAS

1.3 Temporal Properties

Motivating Example: Voting

Properties to express

- The voting system will not reveal how a particular voter voted
- The voter will eventually cast a vote
- The voter can vote, and can refrain from voting

Logical Specification of Properties

Specification of Temporal Properties

Temporal logic: mathematical logic with additional operators to describe how the system will (or may) evolve:

$\bigcirc\varphi$	φ is true in the next moment in time
$\Box\varphi$	φ is true in all future moments
$\Diamond\varphi$	φ is true in some future moment
$\varphi\,\mathrm{U}\,\psi$	φ is true until the moment when ψ becomes true

Specification Templates

Temporal logic has achieved a significant role in the formal specification and verification of concurrent and distributed systems.

Much of the popularity was achieved because some useful concepts can be formally, and concisely, specified using temporal logics, e.g.:

- safety properties
- reachability properties
- fairness properties

Specification Templates: Safety Properties

Safety / maintenance:

"something bad will never happen"
"something good will always hold"

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Fairness / service:

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$$\Box(\mathrm{needMoney} \rightarrow \Diamond \mathrm{getMoney})$$

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Usually: $\Box\Diamond \ldots$

Formal Semantics of Linear Time Logic (LTL)

Definition 1.1 (Semantics of LTL)

$M, \lambda \models p$ iff p is true at moment $M, \lambda[0]$;

$M, \lambda \models \bigcirc\varphi$ iff $M, \lambda[1..\infty] \models \varphi$;
$M, \lambda \models \Diamond\varphi$ iff $M, \lambda[i..\infty] \models \varphi$ for some $i \geq 0$;
$M, \lambda \models \Box\varphi$ iff $M, \lambda[i..\infty] \models \varphi$ for all $i \geq 0$;
$M, \lambda \models \varphi \, \mathrm{U} \, \psi$ iff $M, \lambda[i..\infty] \models \psi$ for some $i \geq 0$, and $M, \lambda[j..\infty] \models \varphi$ for all $0 \leq j < i$.

$M, \lambda \models \neg\varphi$ iff not $M, \lambda \models \varphi$;
$M, \lambda \models \varphi \wedge \psi$ iff $M, \lambda \models \varphi$ and $M, \lambda \models \psi$.

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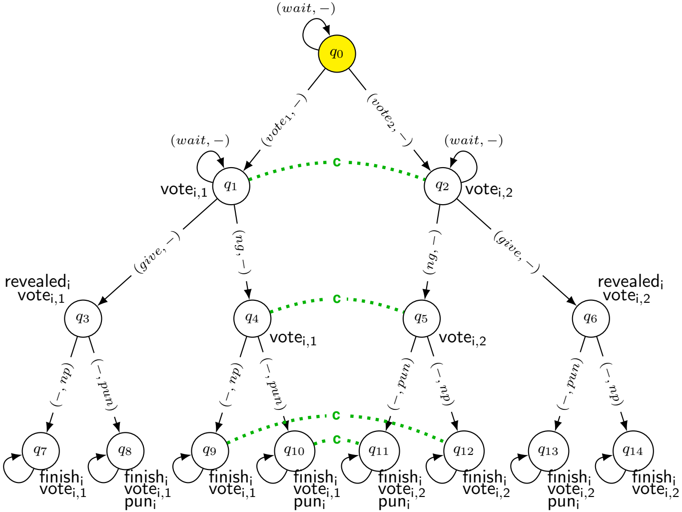

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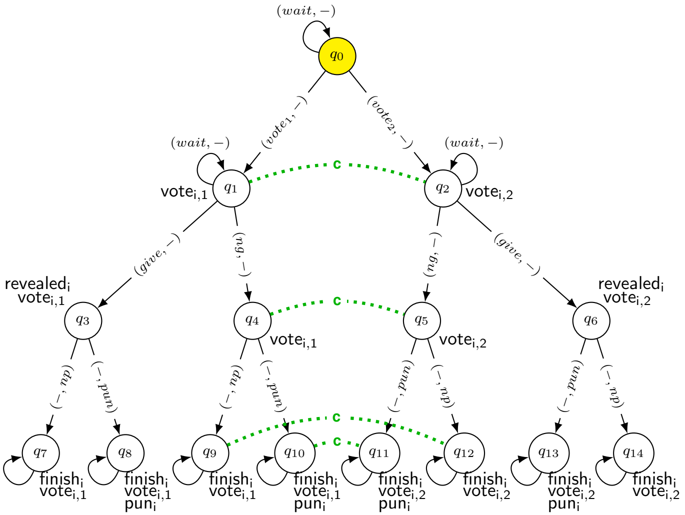

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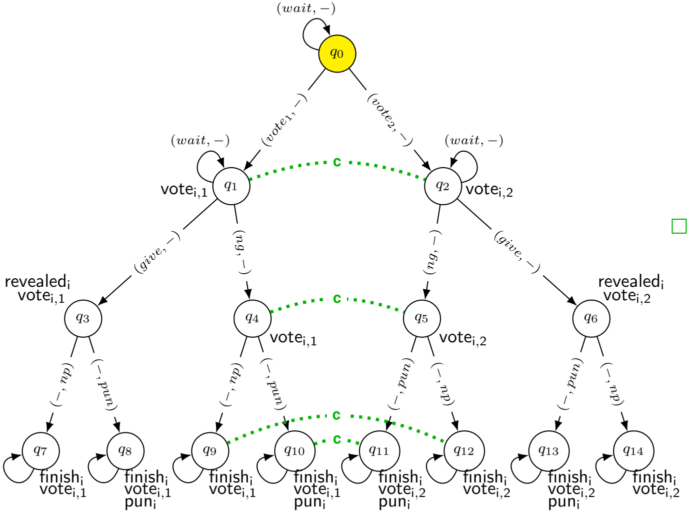

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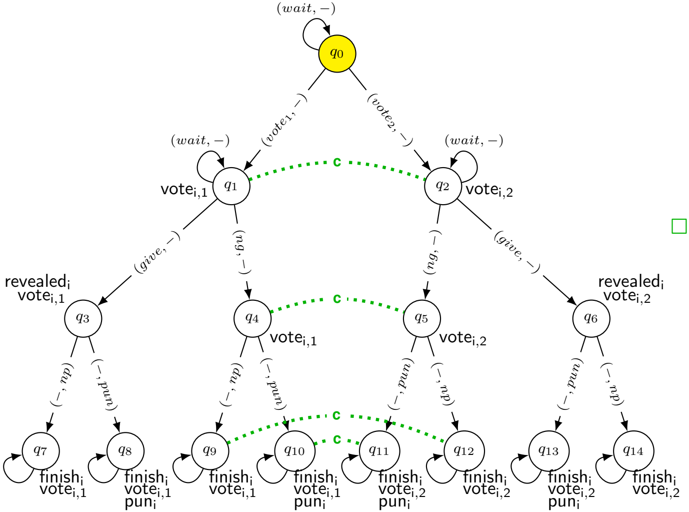

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  Cannot be expressed in LTL

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1.4 Strategic Abilities

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- In multi-agent systems, it is often very important to know **who** can **make them evolve** in a particular way

Motivating Example: Voting

Properties to express

Privacy: The system cannot reveal how a particular voter voted

Enfranchisement: The voter can vote, and can refrain from voting

Coercion-resistance: The voter cannot convince the coercer that she voted in a certain way

Logical Specification of Strategic Abilities

- ATL: Alternating-time Temporal Logic [Alur et al. 1997-2002]
- Temporal logic meets game theory
- Main idea: cooperation modalities

$\langle\langle A \rangle\rangle \Phi$: coalition A has a collective strategy to enforce Φ

$\leadsto$ Φ can include temporal operators: $\bigcirc$ (next), $\Diamond$ (sometime in the future), $\Box$ (always in the future), U (strong until)

Example Formulas

- $\langle\langle jamesbond \rangle\rangle \Diamond (\mathsf{ski} \wedge \neg \mathsf{getBurned})$:
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- $\langle\langle jamesbond, bondsgirl \rangle\rangle (\neg \mathsf{destruction})\, \mathrm{U}\, \mathsf{endOfMovie}$:
  "James Bond and his girlfriend are able to save the world from destruction until the end of the movie"

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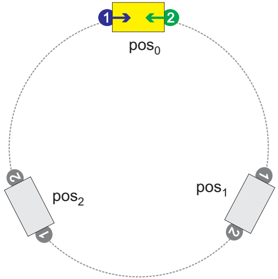

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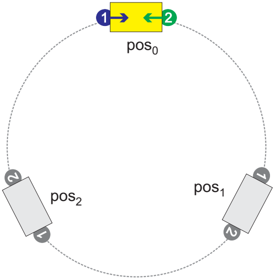

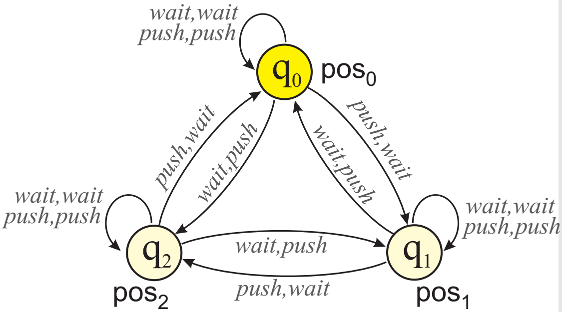

Strategies and Abilities

Strategy

A **strategy** is a conditional plan.
We represent strategies by functions $s_a : St \to Act$.

$\leadsto$ memoryless strategies

Alternative: perfect recall strategies $s_a : St^+ \to Act$

Semantics of ATL

$M, q \models \langle\langle A \rangle\rangle \Phi$ iff there is a collective strategy s_A such that, for every path λ that may result from execution of s_A from q on, we have that $M, \lambda \models \Phi$.

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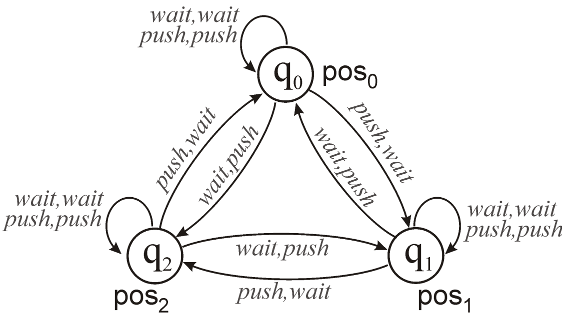

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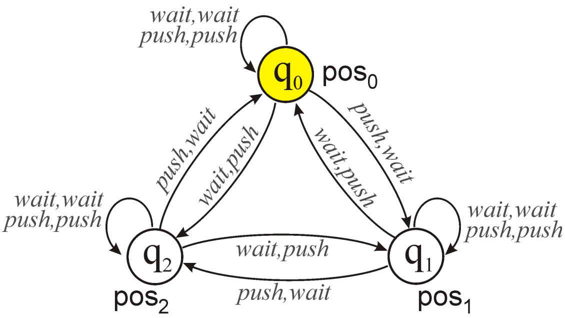

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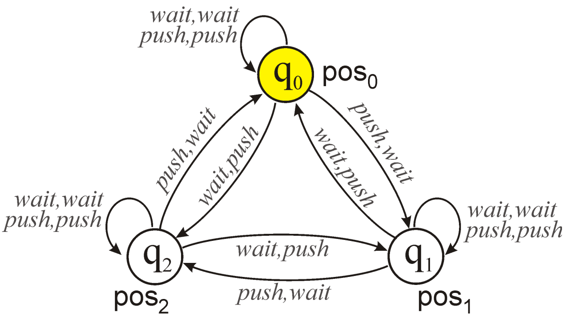

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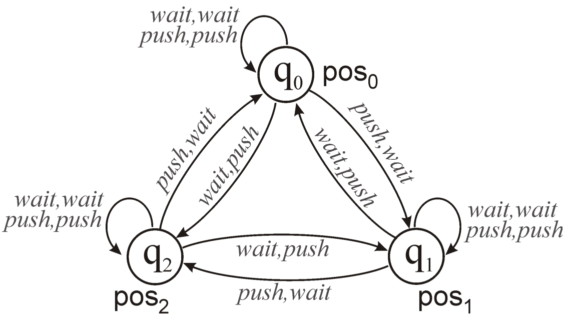

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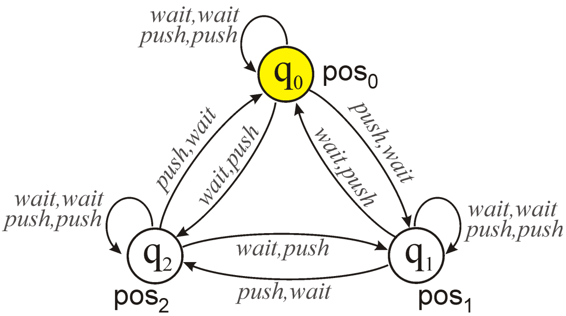

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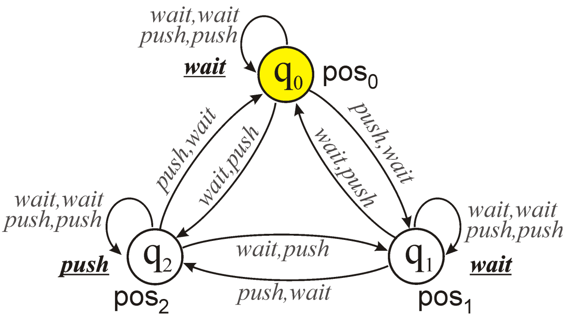

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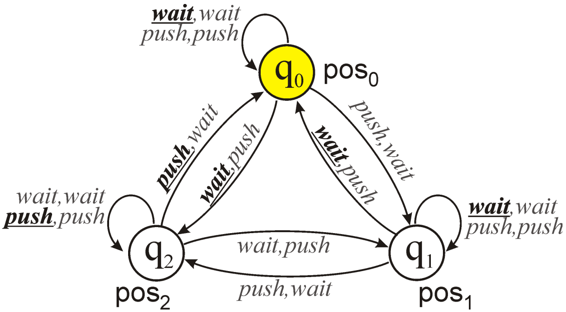

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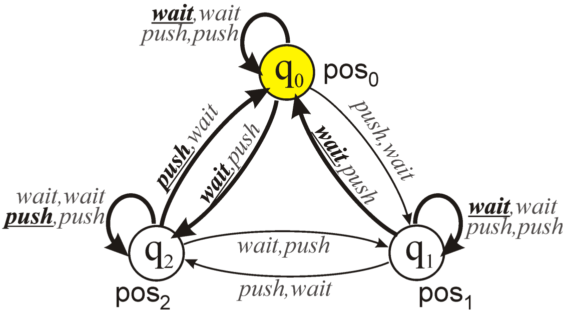

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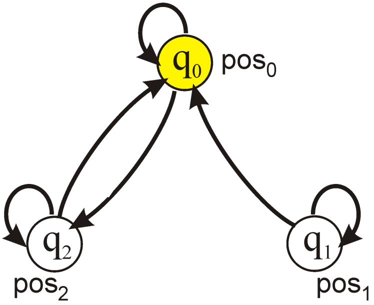

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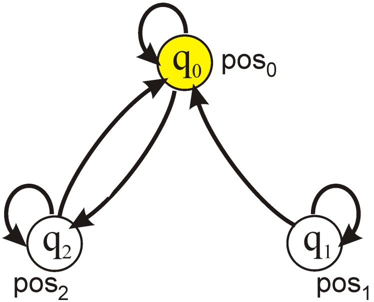

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Yes!

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1.5 Imperfect Information

Executable Strategies under Imperfect Information

Strategies under imperfect information must be executable $\rightsquigarrow$ **uniform strategies**

Definition 1.2 (Uniform strategy)

Strategy s_a is **uniform** iff it specifies the same choices for indistinguishable situations:

- (no recall:) if $q \sim_a q'$ then $s_a(q) = s_a(q')$
- (perfect recall:) if $h \approx_a h'$ then $s_a(h) = s_a(h')$
  where $h \approx_a h'$ iff $h[i] \sim_a h'[i]$ for every i.

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A collective strategy is uniform iff it consists only of uniform individual strategies.

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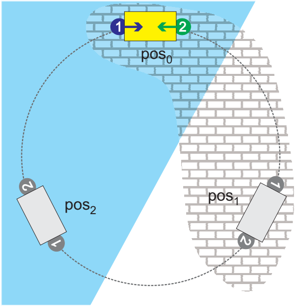

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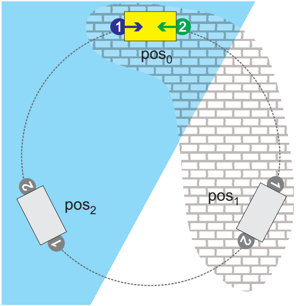

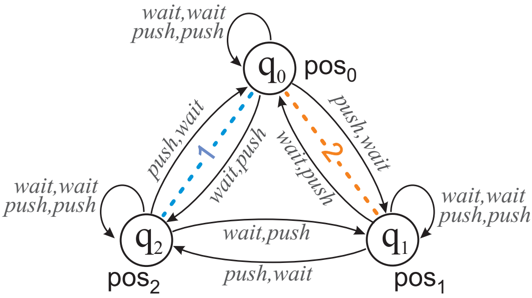

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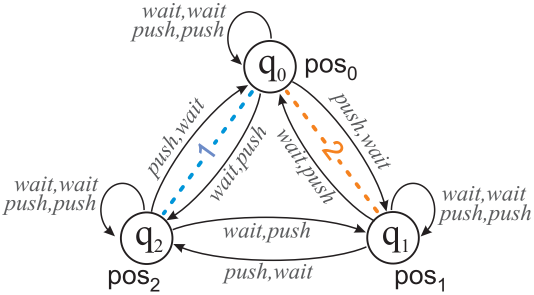

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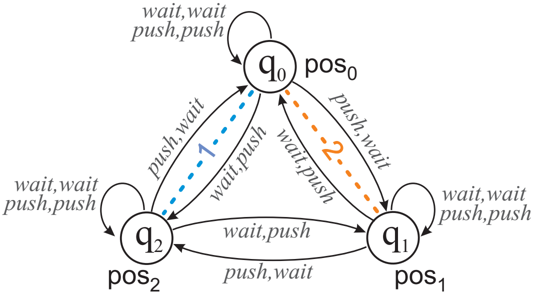

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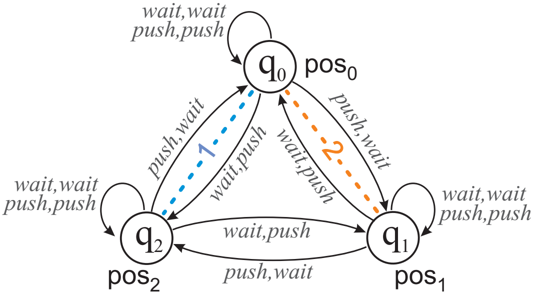

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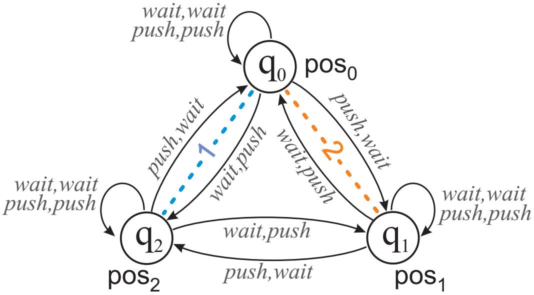

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Strategies and Knowledge

Note:

Having a successful strategy does not imply knowing that we have it!

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Knowing that a successful strategy exists does not imply knowing the strategy itself!

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- The voter can vote, and can refrain from voting

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  Cannot be expressed in ATL (we need a notion of knowledge for the coercer) !

Part 1: Introduction to Model Checking for MAS

1.6 Adding Knowledge Operators

Adding Knowledge Operators

- Epistemic operators: $K_i\varphi$ ("i knows that φ")
- Semantics: φ holds in all the states that look the same as the current state to i

$M, q \models K_i\varphi$ iff $M, q' \models \varphi$ for all q' such that $q \sim_i q'$

Collective Knowledge

A group of agents A can know that φ in several different epistemic modes:

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- $D_A\varphi$: A have distributed knowledge that φ

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  Better specification: $\bigwedge_{j \in \mathit{Cand}} \neg \langle\langle i, c \rangle\rangle \Diamond K_c \mathsf{voted}_{i,j}$

  Even better: $\bigwedge_{C \subsetneq \mathit{Cand}} \neg \langle\langle i, c \rangle\rangle \Diamond K_c \big(\bigvee_{j \in C} \mathsf{voted}_{i,j} \big)$

Part 1: Introduction to Model Checking for MAS

1.7 Model Checking

Verification by Model Checking

Model checking problem

$$M, q \overset{?}{\models} \varphi$$

transition system modal formula

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transition system   modal formula

That is, we want to implement function $mcheck(M, q, \varphi)$ such that:

$$mcheck(M, q, \varphi) = \begin{cases} \top & \text{if } \quad M, q \models \varphi \\ \bot & \text{else} \end{cases}$$

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This problem is sometimes called **local model checking**

Local vs. Global Model Checking

Local model checking

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Often no harder than local model checking...

Part 2: Verification of Strategic Ability

Model Checking ATL

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2.1 Fixpoint Algorithm

Fixpoint Algorithm for ATL with Perfect Information

A well-known nice result: model checking ATL for agents with perfect information is tractable!

Theorem (Alur, Kupferman & Henzinger 1998/2002)

Model checking ATL with perfect information is P-complete, and can be done in linear time.

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Model checking ATL with perfect information is P-complete, and can be done in time $O(ml)$ where $m =$ #transitions in the model and $l =$ #symbols in the formula.

Fixpoint Algorithm for ATL with Perfect Information

The algorithm is based on the following **fixpoint equivalences**:

- $\langle\langle A\rangle\rangle\Box\varphi \quad \leftrightarrow \quad \varphi \wedge \langle\langle A\rangle\rangle \bigcirc \langle\langle A\rangle\rangle\Box\varphi$

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- $\langle\langle A\rangle\rangle\varphi_1\,\mathrm{U}\,\varphi_2 \quad\leftrightarrow\quad \varphi_2\vee\varphi_1\wedge\langle\langle A\rangle\rangle\bigcirc\langle\langle A\rangle\rangle\varphi_1\,\mathrm{U}\,\varphi_2$.

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Perfect information strategies for reachability/safety objectives can be synthesized incrementally (no backtracking is necessary).

function $mcheck(\mathcal{M}, \varphi)$**.**
Global model checking formulae of ATL.
Returns the exact subset of St for which formula φ holds.

case $\varphi \equiv p$ **:** return $V(p)$
case $\varphi \equiv \neg\psi$ **:** return $St \setminus mcheck(\mathcal{M}, \psi)$
case $\varphi \equiv \psi_1 \wedge \psi_2$ **:** return $mcheck(\mathcal{M}, \psi_1) \cap mcheck(\mathcal{M}, \psi_2)$
case $\varphi \equiv \langle\langle A \rangle\rangle \bigcirc \psi$ **:** return $pre(A, mcheck(\mathcal{M}, \psi))$
case $\varphi \equiv \langle\langle A \rangle\rangle \Box \psi$ **:**
$Q_1 := Q$; $Q_2 := Q_3 := mcheck(\mathcal{M}, \psi)$;
while $Q_1 \not\subseteq Q_2$ **do** $Q_1 := Q_1 \cap Q_2$; $Q_2 := pre(A, Q_1) \cap Q_3$ **od**;
return Q_1
case $\varphi \equiv \langle\langle A \rangle\rangle \psi_1 \, \mathrm{U} \, \psi_2$ **:**
$Q_1 := \emptyset$; $Q_2 := mcheck(\mathcal{M}, \psi_2)$; $Q_3 := mcheck(\mathcal{M}, \psi_1)$;
while $Q_2 \not\subseteq Q_1$ **do** $Q_1 := Q_1 \cup Q_2$; $Q_2 := pre(A, Q_1) \cap Q_3$ **od**;
return Q_1
end case

$$\text{pre}(A, Q) = \{q \mid \exists \alpha_A \forall \alpha_{\mathbb{A}\mathrm{gt} \setminus A}\, o(q, \alpha_A, \alpha_{\mathbb{A}\mathrm{gt} \setminus A}) \in Q\}$$

Example: Simple Rocket Domain

- Assume that there are 3 workers in the rocket (agents 1, 2, and 3)

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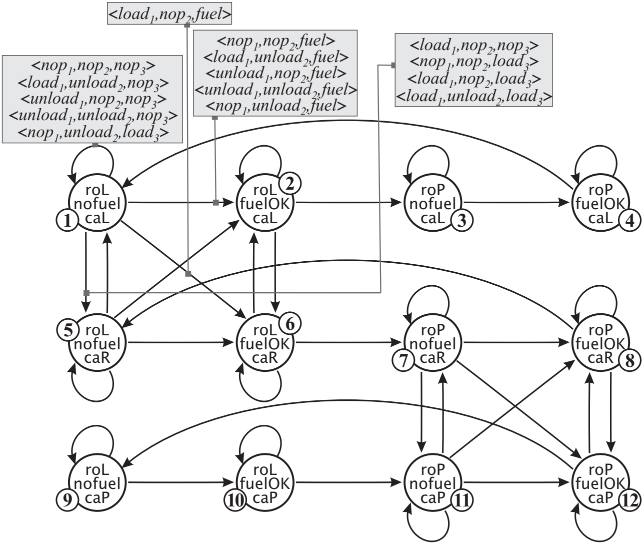
- Flying has highest priority: if agent 1 initiates the flight, current actions of the other agents have no effect
- If loading is attempted when the cargo is not around, nothing happens
- Same for unloading when the cargo is not in the rocket, and refilling a full tank
- If different agents try to load and unload at the same time then the majority prevails
- Refilling fuel can be done in parallel with loading/unloading

Example: Simple Rocket Domain



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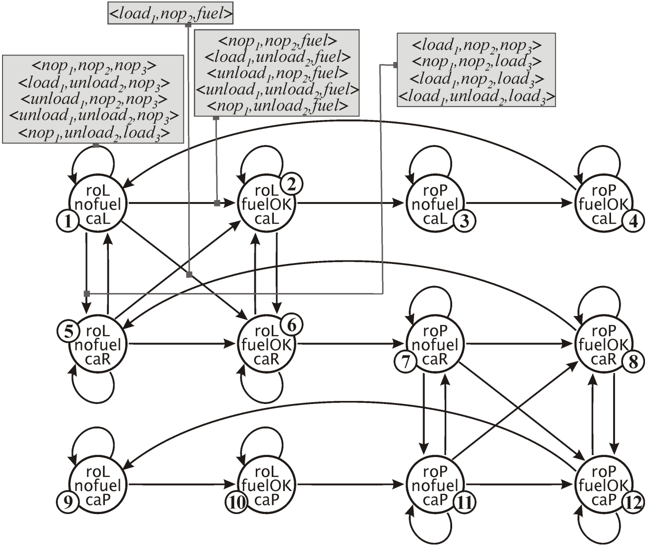
- How does that work for the coalition of agents 1 and 2 ($\langle\langle 1,2\rangle\rangle\Diamond\mathsf{caP}$) ?
- What about a maintenance goal, like agent 3 keeping the cargo in Paris forever ($\langle\langle 3\rangle\rangle\Box\mathsf{caP}$) ?

Simple Rocket Domain: Verification of $\langle\langle 1,3\rangle\rangle\Diamond$caP



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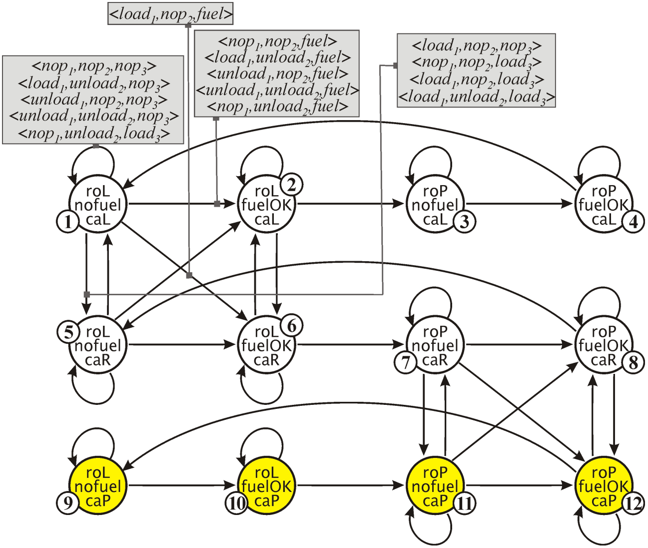

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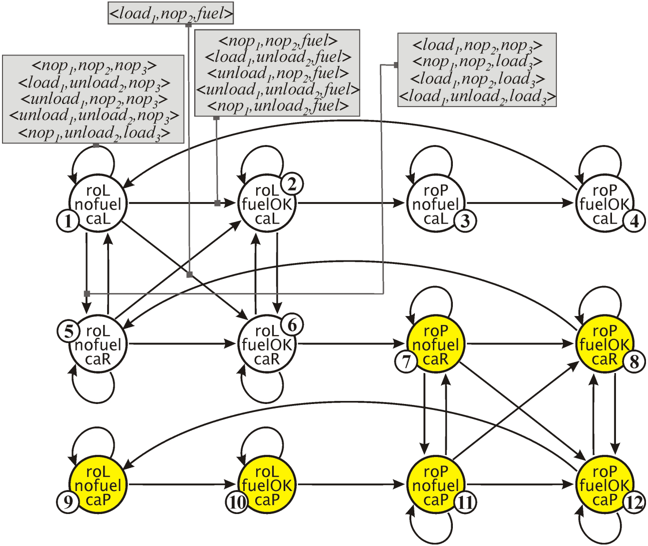

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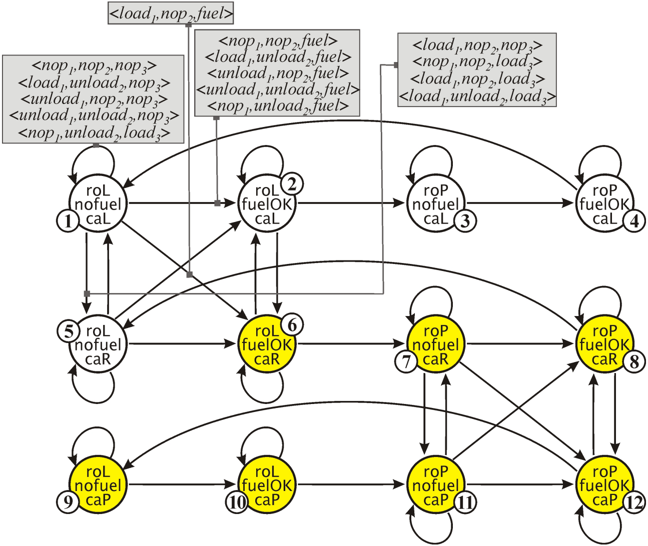

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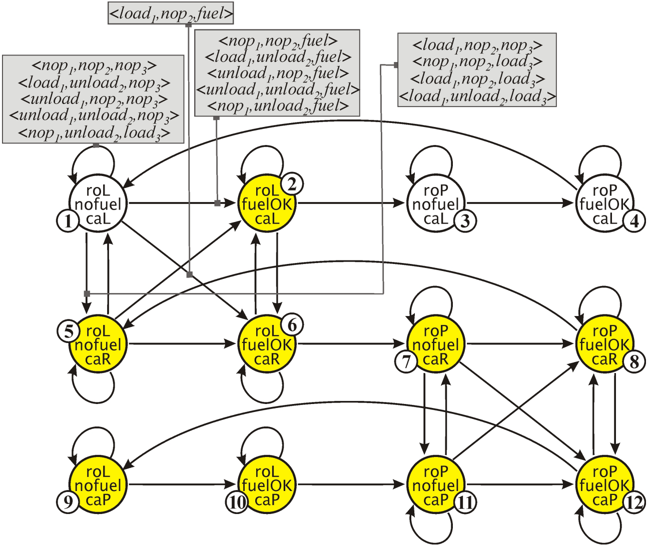

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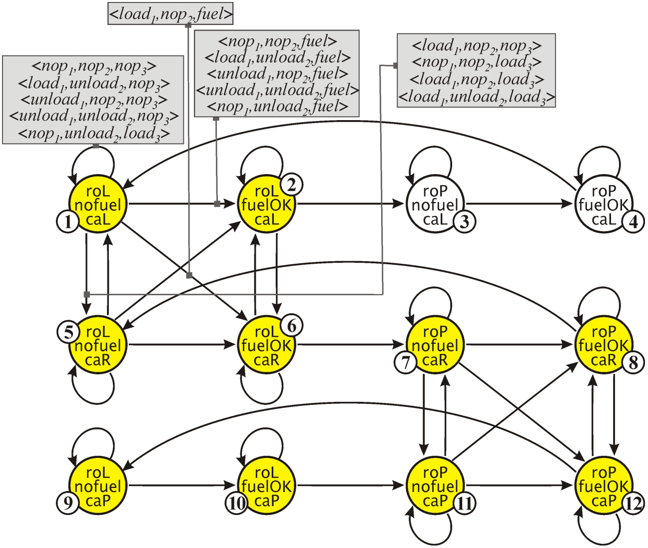

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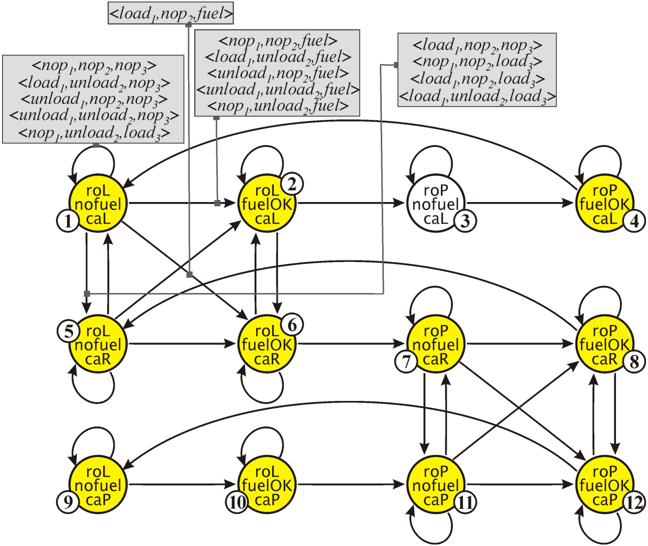

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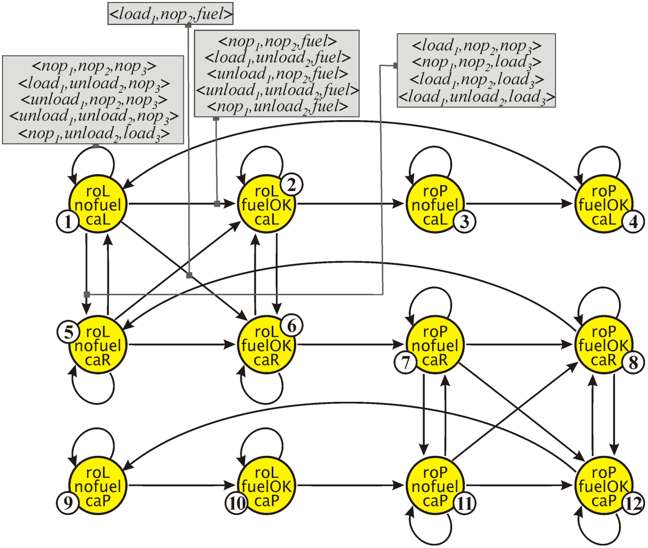

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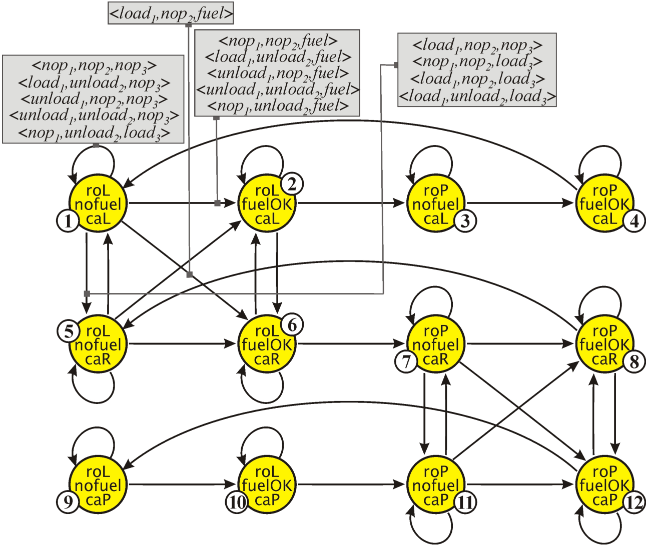
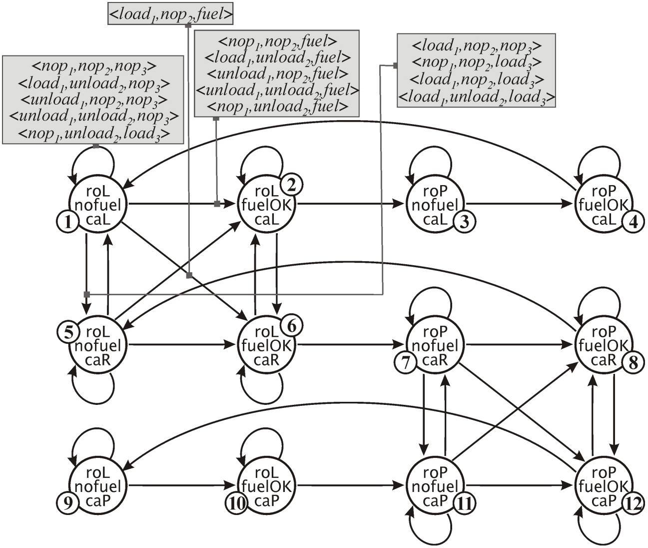
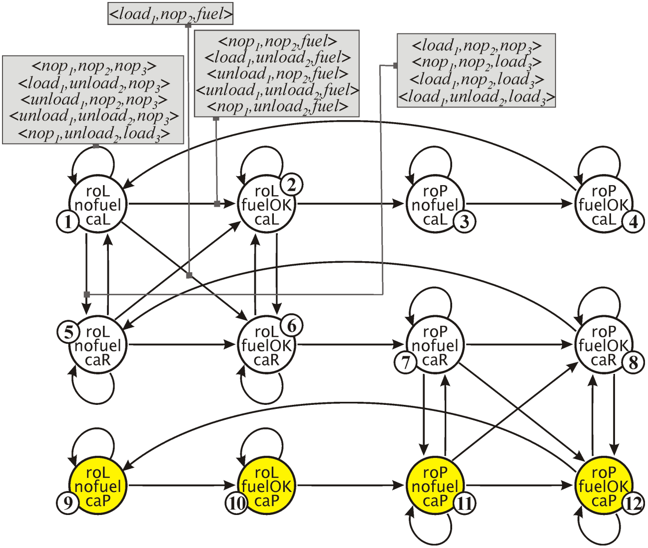


Done!

Simple Rocket Domain: Verification of $\langle\langle 1,2\rangle\rangle\Diamond$caP



Simple Rocket Domain: Verification of $\langle\langle 1, 2\rangle\rangle\Diamond$caP



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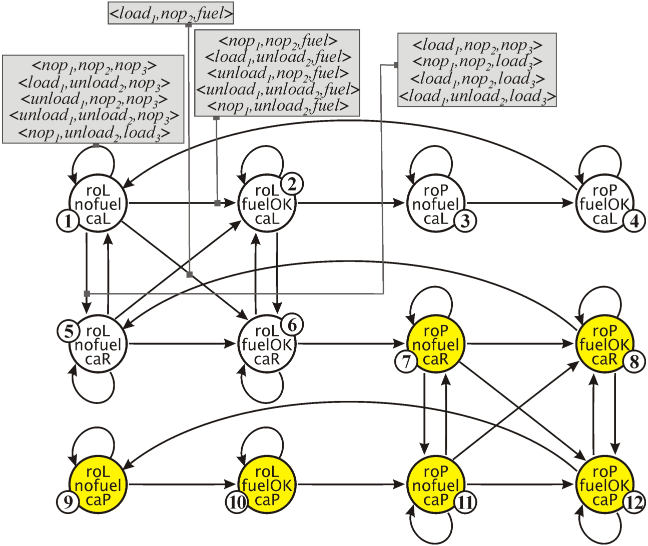

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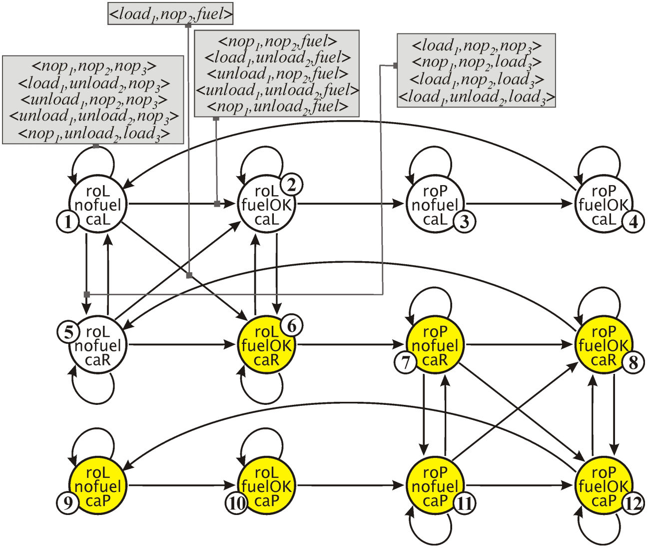

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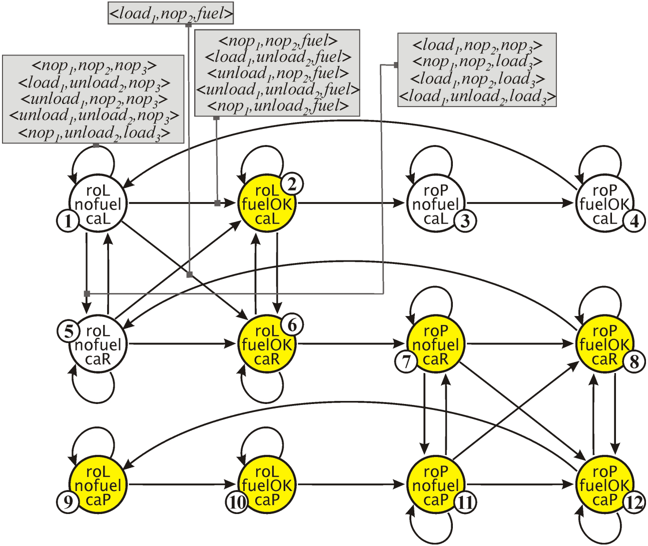

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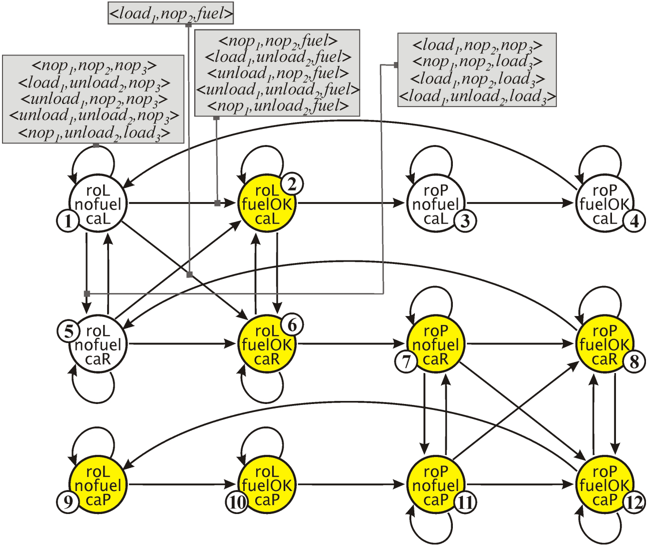
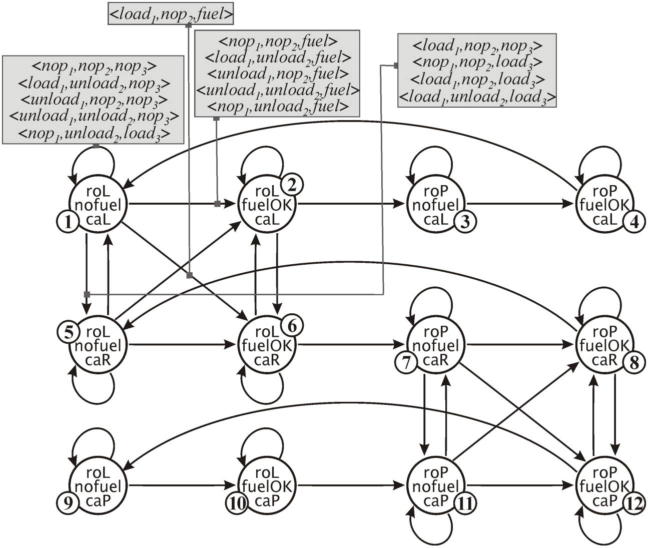


Done!

Simple Rocket Domain: Verification of $\langle\langle 3\rangle\rangle\Box$caP



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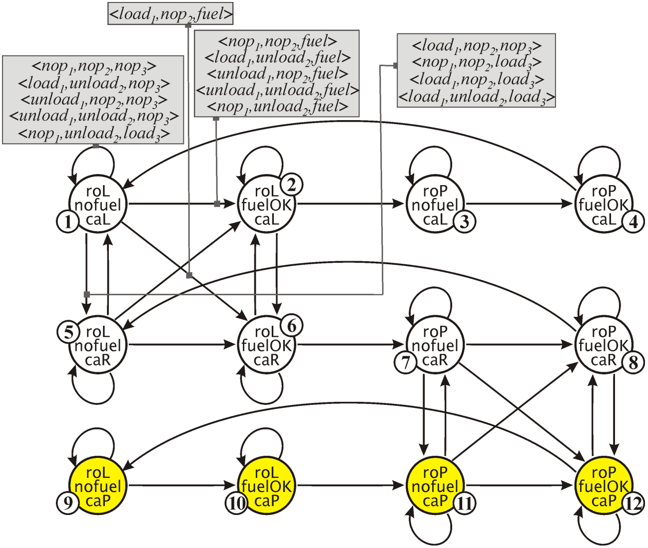

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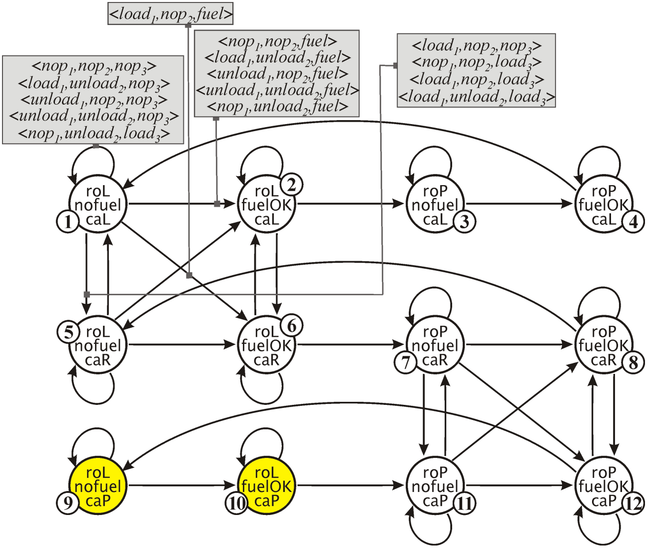

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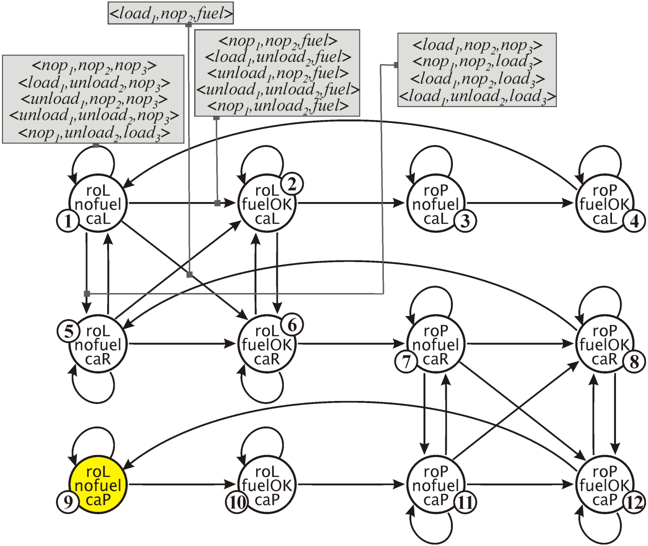

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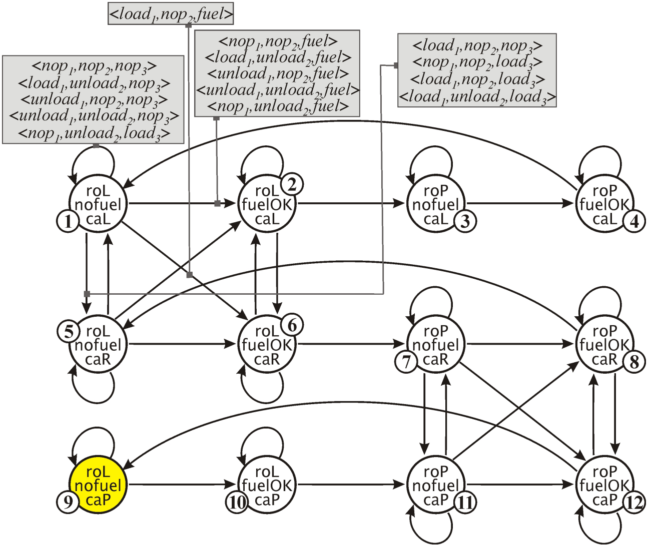

Done!

Model Checking ATL for Perfect Information

Theorem (Alur, Kupferman & Henzinger 1998/2002)

ATL model checking for perfect information games is $\mathbf{P}$-complete, and can be done in linear time.

So... let's model check!

Not That Easy...

Challenges:

- Preparing the model $\rightsquigarrow$ socio-technical system!
- Writing formula for the requirement(s)

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- Preparing the model $\rightsquigarrow$ socio-technical system!
- Writing formula for the requirement(s)

- State- and transition-space explosion
- Invalidity of fixpoint equivalences for imperfect information

Part 2: Verification of Strategic Ability

2.2 Imperfect Information

Model Checking ATL: Imperfect Information

Theorem (Schobbens 2004; Jamroga & Dix 2006)

Model checking ATL for agents with imperfect information playing memoryless strategies is Δ_2-complete in the number of transitions in the model and the length of the formula.

(where $\mathbf{\Delta_2^P}$ is the class of problems solvable in polynomial time by a deterministic Turing machine making adaptive calls to an oracle solving NP problems)

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(where $\mathbf{\Delta_2^P}$ is the class of problems solvable in polynomial time by a deterministic Turing machine making adaptive calls to an oracle solving NP problems)

Corollary

Imperfect information strategies cannot be synthesized incrementally: we cannot do better than guess the whole strategy and check if it succeeds.

Model Checking ATL: Imperfect Info, Perfect Recall

What about agents with perfect recall and imperfect information?
The news are bad...

Theorem (Dima and Tiplea, 2011)

Model checking ATL for agents with imperfect information and perfect recall is **undecidable**.

The problem is undecidable even for turn-based models with 3 players, and flat formulae with only doubleton coalitions.

It Really Takes Two (To Make Things Undecidable)...

Theorem (Guelev, Dima, and Enea, 2010)

Model checking ATL for singleton coalitions with imperfect information and perfect recall is **EXPTIME-complete**.

Not Easy Indeed...

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- Possible way out: **incomplete algorithms**

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- Note: the main source of complexity is the size of the model!
- Possible way out: use smaller models $\rightsquigarrow$ **model reductions**

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- Note: the main source of complexity is the size of the model!
- Possible way out: use smaller models $\rightsquigarrow$ **model reductions**

- Also, we will only look at memoryless strategies from now on

Part 3: Practical Model Checking

Towards Practical Model Checking for Strategies

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Two ideas:

- Approximate model checking
- Brute force search with local optimization

Part 3: Practical Model Checking

3.1 Approximate Model Checking

Approximate Verification of Strategic Ability

- Exact verification of strategic abilities is hard
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- Idea: try to find formulae that **approximate the truth value of the given specification** (i.e., upper bound and lower bound)
- ...and which are easier to compute
- If lower bound = upper bound, we get the exact answer!

Approximate Verification of Strategic Ability

Approximation Semantics

$$
\begin{aligned}
&LB(p) = p,\\
&LB(\neg\phi) = \neg UB(\phi),\\
&LB(\phi \wedge \psi) = LB(\phi) \wedge LB(\psi),\\
&LB(\langle A\rangle\phi) = \langle A\rangle LB(\phi),\\
&LB(\langle\langle A\rangle\rangle \Box\phi) = \nu Z.(C_A LB(\phi) \wedge \langle A\rangle^{\bullet} Z),\\
&LB(\langle\langle A\rangle\rangle \psi \,\mathrm{U}\, \phi) = \mu Z.\big(E_A LB(\phi) \vee (C_A LB(\psi) \wedge \langle A\rangle^{\bullet} Z)\big).
\end{aligned}
$$

$$
\begin{aligned}
&UB(p) = p,\\
&UB(\neg\phi) = \neg LB(\phi),\\
&UB(\phi \wedge \psi) = UB(\phi) \wedge UB(\psi),\\
&UB(\langle A\rangle\phi) = E_A \langle\langle A\rangle\rangle_{\mathrm{Ir}} \bigcirc UB(\phi),\\
&UB(\langle\langle A\rangle\rangle \Box\phi) = E_A \langle\langle A\rangle\rangle_{\mathrm{Ir}} \Box\, UB(\phi),\\
&UB(\langle\langle A\rangle\rangle \psi \,\mathrm{U}\, \phi) = E_A \langle\langle A\rangle\rangle_{\mathrm{Ir}}\, UB(\psi) \,\mathrm{U}\, UB(\phi).
\end{aligned}
$$

Approximate Verification of Strategic Ability

Theorem (Jamroga, Knapik, Kurpiewski, & Mikulski 2019)

For every pointed model M and ATL formula φ:

$$M \models LB(\varphi) \implies M \models \varphi \implies M \models UB(\varphi).$$

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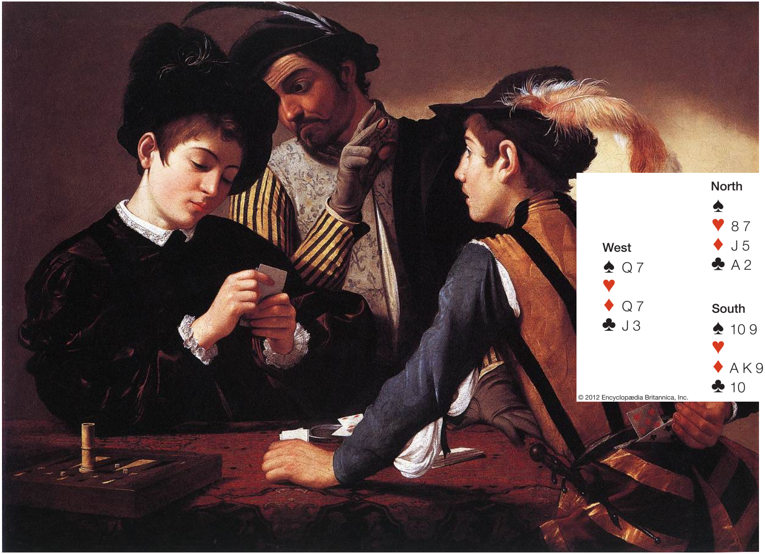
- Benchmark: card play (similar mathematical structure to coercion in a voting protocol!)

Approximate Verification of Strategic Ability

Approximate Verification of Strategic Ability



Experimental Results

#cards	#states	Approximate verification				Exact verif.
		tgen	lower	upper	match	
4	11	$<$0.01	$<$0.01	$<$0.01	100%	0.12
8	346	0.01	$<$0.01	$<$0.01	100%	2.42 h*
12	12953	0.7	0.07	0.01	100%	timeout
16	617897	35.2	348.4	0.7	100%	timeout
$20^{\star}$	2443467	132.0	8815.7	4.2	100%	timeout

Formula: $\langle\langle \mathbf{S} \rangle\rangle \Diamond \mathsf{win}$

Time in seconds, unless explicitly indicated
timeout $\approx$ 45h

Experimental Results with Optimized Data Structures

#cards	#states	Approximate verification				Exact verif.
		tgen	lower	upper	match	
4	11	<0.01	<0.01	<0.01	100%	0.12
8	346	<0.01	<0.01	<0.01	100%	2.42 h*
12	12953	0.06	<0.01	<0.01	100%	timeout
16	617897	4.6	0.6	0.3	100%	timeout
20*	2443467	34.0	3.0	2.0	100%	timeout
20	1.5 e7	124.0	8.5	6.0	100%	timeout
24*	7 e7	3779.0	667.0	78.0	100%	timeout

Formula: $\langle\langle \mathbf{S} \rangle\rangle \Diamond \mathsf{win}$

Time in seconds, unless explicitly indicated
timeout $\approx$ 45h

Experimental Results for Absent-Minded Declarer

#cards	#states	Approximate verification				Exact verification
		tgen	lower	upper	match	
4	19	<0.01	<0.01	<0.01	100%	9.68 h*
8	713	0.04	0.01	<0.01	100%	timeout
12	52843	5.2	18.6	0.6	80%	timeout
16	memout					timeout

Formula: $\langle\langle \mathbf{S} \rangle\rangle \Diamond \mathsf{win}$

Time in seconds, unless explicitly indicated
timeout $\approx$ 45h

Part 3: Practical Model Checking

3.2 DominoDFS

DominoDFS

- When no better idea, try brute-force search for a winning strategy
- Give luck a chance $\rightsquigarrow$ Depth-First Search (DFS)

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DominoDFS

- When no better idea, try brute-force search for a winning strategy
- Give luck a chance $\rightsquigarrow$ Depth-First Search (DFS)
- Idea: optimize the search by discarding dominated partial strategies
- Additional advantage: might work where fixpoint approximation is guaranteed to fail

Strategic Domination

- Consider a partial strategy σ_a defined in an epistemic class $[q]_{\sim_a}$
- The context of σ_a is given by a partial (possibly nondeterministic) strategy σ_a^C defined everywhere outside $[q]_{\sim_a}$
- Then, (σ_a, σ_a^C) specify a full (possibly nondeterministic) strategy of agent a

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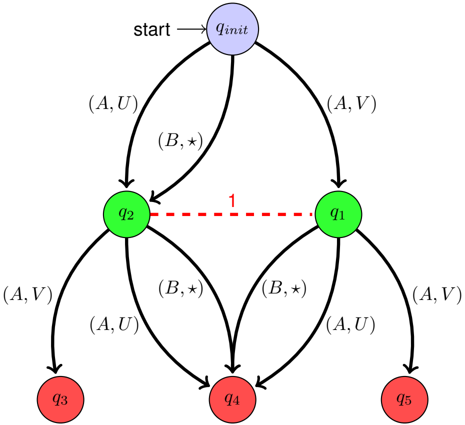
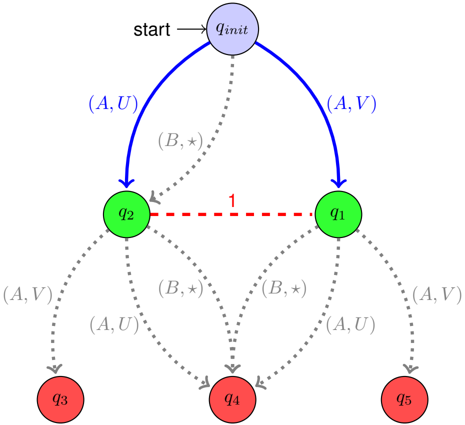
Strategic Domination

Partial strategy σ_a **dominates** σ'_a with respect to context σ_a^C iff the outcome paths of (σ'_a, σ_a^C) strictly subsume those of (σ_a, σ_a^C).

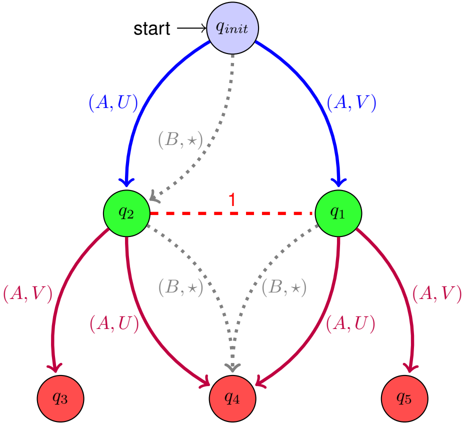
Domination-Based Depth-First Strategy Search



Domination-Based Depth-First Strategy Search



Domination-Based Depth-First Strategy Search



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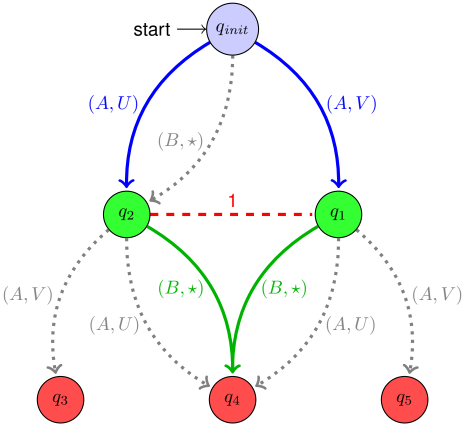
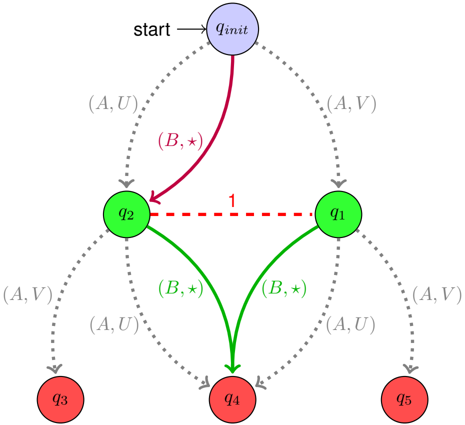


Domination-Based Depth-First Strategy Search



Experimental Results: Bridge Endplay

#cards	DominoDFS	MCMAS	Approx.	Approx. opt.
4	0.0006	0.12	0.0008	< 0.0001
8	0.01	8712	0.01	< 0.0001
12	0.8	timeout	0.8	0.06
16	160	timeout	384	5.5
20	1373	timeout	8951	39
24	memout	timeout	memout	4524

Experimental Results: Castles

#agents	DominoDFS	MCMAS	brute force (SMC)
$(1,1,1)$	0.3	65	63
$(2,1,1)$	1.5	12898	184
$(2,2,1)$	25	timeout	4923
$(2,2,2)$	160	timeout	timeout
$(3,2,2)$	2688	timeout	timeout
$(3,3,2)$	timeout	timeout	timeout

Fixpoint approximation bound to be inconclusive!

Part 4: Model Reductions

Model Reductions

Factors of complexity:

- Size of the model $\rightsquigarrow$ “program complexity”
- Length of the formula $\rightsquigarrow$ “formula complexity”

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Want to make the verification feasible? Use smaller models!

$\rightsquigarrow$ **Model reductions**

Model Reductions

Model Reductions

Part 4: Model Reductions

4.1 Bisimulation-Based Reduction

Bisimulation-Based Model Reduction

Given are:

- M, M': two iCGS's, sharing the set of agents $\mathbb{A}\mathrm{gt}$ and the set of atoms AP
- coalition $A \subseteq \mathbb{A}\mathrm{gt}$
- relation $\Rrightarrow_A \subseteq S \times S'$ between the states of M and M'.

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Strategy simulator

A simulator of partial strategies for coalition A with respect to $\Rrightarrow_A$ is any family of functions

$$ST = \{ST_{C_A(q),C_A(q')} : PStr_A(C_A(q)) \to PStr_A(C_A(q')) \quad | \quad q \Rrightarrow_A q'\}.$$

The idea is that $ST_{C_A(q),C_A(q')}$ "transforms" each partial strategy σ_A that works on the neighborhood of q in model M into a corresponding strategy σ'_A that works on the neighborhood of q' in model M'.

Bisimulation-Based Model Reduction

Simulation for ATL$_{ir}$

$\Rrightarrow_A \subseteq S \times S'$ is a **simulation for A** iff there exists a simulator of partial strategies ST such that $q \Rrightarrow_A q'$ implies the following:

1. $\pi(q) = \pi'(q')$;
2. For every $i \in A$ and $r' \in S'$, if $q' \sim'_i r'$ then for some $r \in S$ we have that $q \sim_i r$ and $r \Rrightarrow_A r'$.
3. For any states $r \in C_A(q)$ and $r' \in C'_A(q')$ such that $r \Rrightarrow_A r'$, every partial strategy $\sigma_A \in PStr_A(C_A(q))$, and every state $s' \in succ(r', ST(\sigma_A))$, there exists a state $s \in succ(r, \sigma_A)$ such that $s \Rrightarrow_A s'$.

Bisimulation for ATL$_{ir}$

A relation $\Lleftarrow\!\!\Rrightarrow_A$ is a **bisimulation for A** iff both $\Rrightarrow_A$ and $\Rrightarrow_A^{-1} = \{(q', q) \mid q \Rrightarrow_A q'\}$ are simulations.

Bisimulation-Based Model Reduction

Preservation Theorem for ATL$_{ir}$ (Belardinelli, Condurache, Dima, Jamroga, & Knapik 2021)

If $\Leftrightarrow_A$ is a bisimulation for A and $q \Leftrightarrow_A q'$, then for every A-formula φ,

$$(M, q) \models \varphi \quad \text{if and only if} \quad (M', q') \models \varphi.$$

Bisimulation-Based Model Reduction

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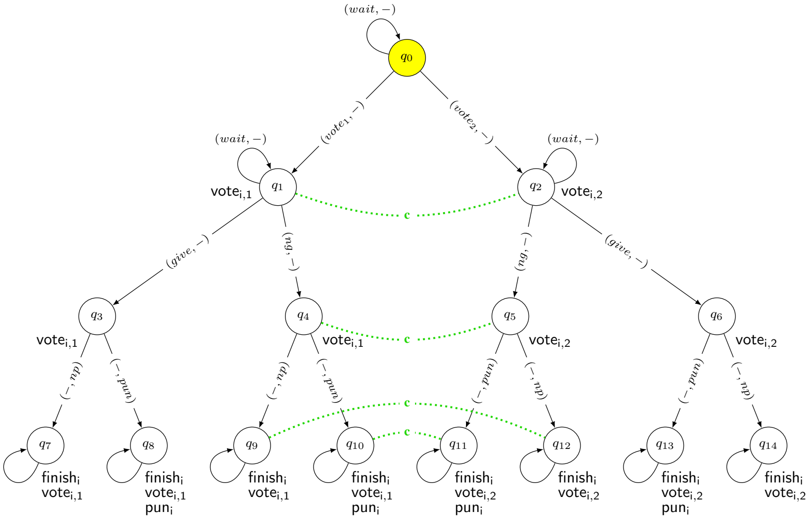
$$(M, q) \models \varphi \quad \text{if and only if} \quad (M', q') \models \varphi.$$

Corollary

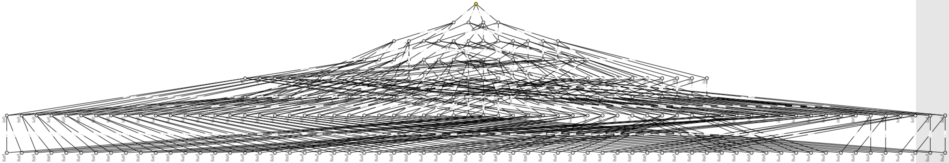
If $\Leftrightarrow$ is a bisimulation for every $A \subseteq \mathbb{A}\mathrm{gt}$, and $q \Leftrightarrow q'$, then for every formula φ,

$$(M, q) \models \varphi \quad \text{if and only if} \quad (M', q') \models \varphi.$$

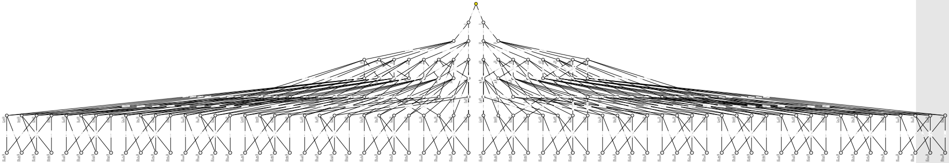
Reduction for Voting and Coercion



Reduction for Voting and Coercion



Reduction for Voting and Coercion



Bisimulation-Based Model Reduction: Summary

- Strong preservation result for the bisimulation (preserves the truth of all ATL_{ir} formulae)
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- ...And the reduced model + bisimulation must be crafted by hand

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- Strong preservation result for the bisimulation (preserves the truth of all ATL_{ir} formulae)
- Can provide very significant reduction **when you know where to look**
- ...But: the conditions are also very strong $\leadsto$ limited applicability
- ...And the reduced model + bisimulation must be crafted by hand
- No methodology/algorithm for **automated reduction**

Part 4: Model Reductions

4.2 Partial Order Reduction

Partial Order Reduction

- **Partial order reduction (POR)**: a method of generating reduced models that preserve the formulae of logic $\mathcal{L}$
- For each infinite path, the reduced model contains at least one $\mathcal{L}$-equivalent path (but as few as possible!)

Partial Order Reduction

- **Partial order reduction (POR)**: a method of generating reduced models that preserve the formulae of logic $\mathcal{L}$
- For each infinite path, the reduced model contains at least one $\mathcal{L}$-equivalent path (but as few as possible!)
- Idea for $LTL_{-\bigcirc}$: take only one arbitrary interleaving of independent actions

Partial Order Reduction for LTL_ $\bigcirc$

Algorithm DFS-POR

A stack represents the path $\pi = g_0 a_0 g_1 a_1 \cdots g_n$ currently being visited.
For g_n, the following three operations are executed in a loop:

1. Compute the set $en(g_n) \subseteq Act$ of enabled actions.
2. Select (heuristically) a subset $E(g_n) \subseteq en(g_n)$ of necessary actions.
3. For any action $a \in E(g_n)$, compute the successor state g' of g_n such that $g_n \xrightarrow{a} g'$, and add g' to the stack.
   Recursively proceed to explore the submodel originating at g'.
4. Remove g_n from the stack.

Partial Order Reduction for LTL_ $\bigcirc$

Conditions for selection of $E(g)$

C1 No action $a \in Act \setminus E(g)$ that is dependent on an action in $E(g)$ can be executed before an action in $E(g)$ is executed.

C2 On every cycle in the constructed state graph there is at least one node g for which $E(g) = en(g)$.

C3 Each action in $E(g)$ is invisible, i.e., it does not change $V(g)$.

Partial Order Reduction for LTL$_{-\bigcirc}$

Theorem (Peled 1993)

For every formula φ of LTL$_{-\bigcirc}$:

$$M \models \varphi \qquad \text{iff} \qquad DFS(M) \models \varphi.$$

Partial Order Reduction for LTL_ $\bigcirc$

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What about **ATL**?

It would seem that a much stronger (and hence less useful) reduction is needed, as ATL is much more expressive than LTL...

Surprise!

Partial Order Reduction for Strategic Abilities

Theorem (Jamroga, Penczek, Sidoruk, Dembinski & Mazurkiewicz 2020)

For every formula φ of **ATL**$_{-\bigcirc}$ without nested strategic operators, interpreted over imperfect information strategies:

$$M \models \varphi \quad \text{iff} \quad DFS(M) \models \varphi.$$

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Note also:

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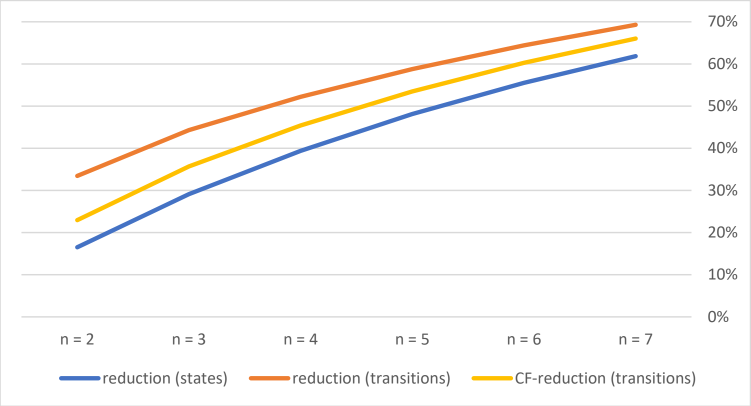
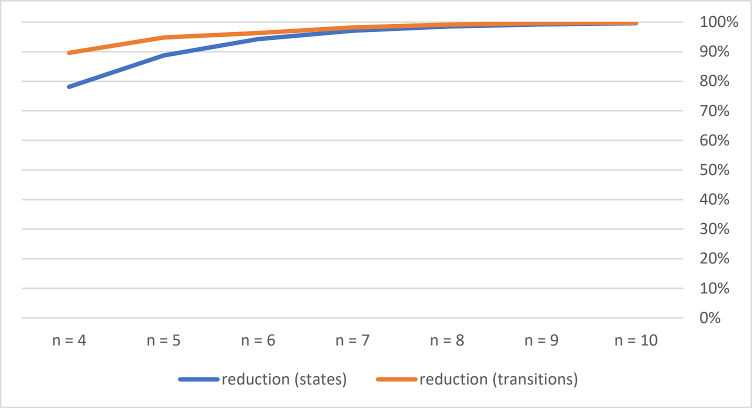
How good are the reductions in practice?

Experimental Results: Asynchronous Simple Voting



Experimental Results: Trains, Gate, and Controller



Partial Order Reduction: Summary

- For some strategic abilities, we get an effective automated model reduction off the shelf for free
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Partial Order Reduction: Summary

- For some strategic abilities, we get an effective automated model reduction off the shelf for free
- There is **free lunch** out there! 
- The trick is... someone must have already paid for the lunch
- What remained was to **prove that we are eligible** to get it (nontrivial!)

Part 5: STrategic Verifier (STV)

Model Checking Tools

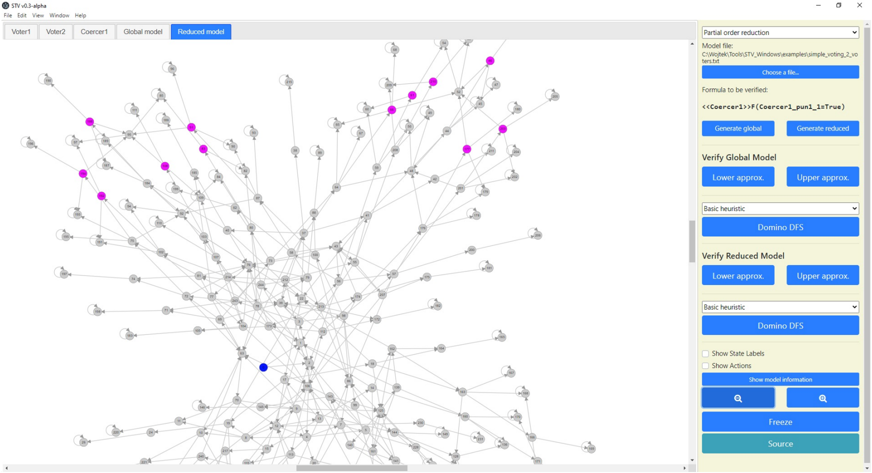
Many model checking tools out there:

- Temporal and timed properties of systems: SPIN, nuSMV, LTSmin, Uppaal, ...
- Temporal-epistemic and strategic properties of MAS: MCMAS (and extensions)
- Strategic properties for imperfect information agents: **STV**

STrategic Verifier (STV)

- Experimental model checker developed at ICS PAS
- Implemented techniques:
  1. standard fixpoint algorithm for perfect info games
  2. brute force DFS
  3. domination-based strategy search
  4. fixpoint approximation
  5. bisimulation checking for bisimulation-based reduction
  6. partial-order reduction
  7. benchmarks and examples
- Includes lightweight GUI and web-based interface

STrategic Verifier (STV)



STrategic Verifier (STV)

Current build: https://github.com/blackbat13/stv

Desktop version for Windows:
https://github.com/blackbat13/stv/releases/download/v0.3.1-alpha/stv-v0.3-alpha-win32-x64.zip

Web version: http://stv.cs-htiew.com/


Thank you