It is declarative
On declarative programming in Prolog

Włodzimierz Drabent

Institute of Computer Science, Polish Academy of Sciences (IPI PAN);
IDA, Linköpings universitet, Sweden
www.ipipan.waw.pl/~drabent/

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This file includes examples, slides and slide overlays not used in the presentation

Logic Programming (LP)

introduced as a declarative programming paradigm

Prolog – implementation of LP

However it seems
the declarative aspect is often neglected, or diminished

Ex.: Now, we compute the factorial usign bottom up method so we start with the trivial problem of computing the factorial of 0 and continue with the factorial of 1, 2 and so on till the factorial of N is known. [...] we [...] store the computed facts using additional parameters. [...] we remember [...] the factorial of M in the M-th step.

```
fact_bu(N,F):-fact_bu1(0,1,N,F).
fact_bu1(N,F,N,F).
fact_bu1(N1,F1,N,F):-
  N1<N, N2 is N1+1, F2 is N2*F1, fact_bu1(N2,F2,N,F).
```

(From a Prolog tutorial [Barták'98])

Declarative descriptions:

% `fact_bu1`(N', F', N, F) – if $0 \leq N' \leq N$ and $F' = N'!$ then $F = N!$

or – $F = F' * (N'+1) * (N'+2) * \cdots * N$

We do not understand a program without understanding the relations it defines.

This talk

A look at the basics of LP

LP in Prolog

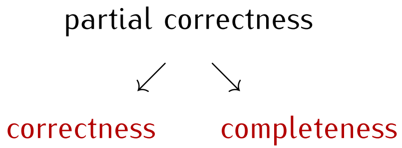
Practical Prolog programming can be declarative

or

Prolog can be used for LP

to an extent larger than usually supposed/understood/meant.

Program correctness in LP



Imperative programming : partial correctness

LP : correctness completeness

correctness = the answers of the program are as required
completeness = all required answers are answers of the program

Df.: correctness$^+$ = correctness + completeness
(full correctness?)
(double correctness?)

Outline – main issues of the talk

Introduction; basic notions.

1. Reasoning (declaratively) about correctness$^+$ of programs.
   Role of approximate specifications.
2. A systematic way of constructing correct$^+$ programs.
   from specifications.
   Limitations of semantics preserving program transformations.
3. Declarative diagnosis (aka. algorithmic debugging) made useful.

Introduction

- Basics of LP
- Specifications
- Correctness and completeness
- Examples
- Approximate specifications

Basics of LP

terminology clash
logic ↔ Prolog
↙

Df.: Query – conjunction $A_1, \ldots, A_n$ of atoms (atomic formulae)

Program – set of clauses $A_0 \leftarrow A_1, \ldots, A_n$,

Answer of a program P – query Q such that $P \models Q$
(correct answer)

computed answer substitution
↓

SLD-resolution – obtaining answers $Q\theta$ from an initial query Q

Computed vs. correct answers?
We do not need to distinguish them

$\leadsto$ soundness and completeness of SLD-resolution

Notation

P a theory	$P \models A$ – A logical consequence of P
S an interpretation	$S \models A$ – A true in S

$\mathcal{HB}$ (Herbrand base) – the set of ground atoms

$\mathcal{M}_P = \{\, A \in \mathcal{HB} \mid P \models A \,\}$ – the least Herbrand model of P

Specifications

LP – relational programming.

A logic programmer has to understand the relations defined by her program.

Specification – should describe for each predicate symbol
a relation on ground terms. So:

Df.: Specification – Herbrand interpretation $S \subseteq \mathcal{HB}$
(i.e. a set of ground atoms).

Ex.:

$$S^0_{\rm member} = \{\, mem(e_i, [e_1, \ldots, e_n]) \in \mathcal{HB} \mid 1 \leq i \leq n \,\}$$

List membership

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The relation for p: $[\![p]\!] = \{\vec{t} \mid p(\vec{t}) \in S\}$

Ex.:

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List membership

Note

crucial for program understanding ↙

Specifications (in LP)
play the role of loop invariants (in imperative programming)
or assertions

“understanding a loop means understanding its invariant”
(maybe without explicitly referring to this notion)
[Furia,Meyer,Velder’14 ACM C.Surveys]
[Dijkstra’??]

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A bit of code:

$\cdots$
$\ldots A[i] \ldots$ $\longleftarrow$ Is i here the number of the last already processed element of A? Or the first unprocessed one?
$\ldots$

On programmers who have not learnt about invariants:
if they understand what they are doing they are relying on some intuitive understanding of the invariant anyway, like Molière’s Mr. Jourdain speaking in prose without knowing it.

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LP: understanding a program = understanding the relations it defines

Correctness$^+$ of programs

Let S – a specification, P – a program.

Df.: P is correct w.r.t. S when $\mathcal{M}_P \subseteq S$.
P is complete w.r.t. S when $S \subseteq \mathcal{M}_P$.

Declarative notions,
independent from any operational semantics

LOGIC + CONTROL works, as
correctness$^+$ independent from CONTROL

Details, answers of correct / complete programs

Non-atomic, non-ground answers

Th.: P correct w.r.t. Q: Q an answer of P $\Rightarrow$ $S \models Q$.
P complete w.r.t. Q: $S \models Q$ $\Rightarrow$ Q an answer of P,
when Q ground, or the alphabet of function symbols infinite, or. . .

Ex.: (the extra conditions at completeness)

Alphabet $\{f/1, a/0\}$, $P = \{p(f(X)).\ \ p(a).\}$, $S = \mathcal{HB} = \mathcal{M}_P$.
P complete w.r.t. S.
$S \models p(Y)$, but $p(Y)$ is not an answer of P.

Examples (specifications, correctness, completeness)

Appending lists

$$S^0_{\rm app} = \{\, app(k,l,m) \in \mathcal{HB} \ \mid k,l,m \text{ are lists, } k\hat{\ }l = m \,\},$$

^ means list concatenation.

(The same in another notation:
$\{\, app([x_1,\ldots,x_k],[y_1,\ldots,y_m],[x_1,\ldots,x_k,y_1,\ldots,y_m]) \in \mathcal{HB} \mid k,m \in \mathbb{N} \,\}$)

Standard program APP: $app(\,[\,],L,L\,).$
$app(\,[H|K],L,[H|M]\,) \leftarrow app(\,K,L,M\,).$

APP complete w.r.t. $S^0_{\rm app}$, but not correct. APP $\models app([\,],6,6)$

APP does not define the list appending relation ($\mathcal{M}_{\rm APP} \neq S^0_{\rm app}$).
(There are even opinions that APP is a wrong program.
It is not, see the next slide.)

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Example (cont'd)

APP correct w.r.t. the following specifications

$$S_{\mathrm{app},1} = \left\{ app(k,l,m) \in \mathcal{HB} \;\middle|\; \begin{array}{l}\text{if } k \text{ and } l \text{ are lists}\\ \text{then } m \text{ is a list}\\ \text{and } k\hat{\ }l = m\end{array} \right\}$$

for list appending

$$S_{\mathrm{app},2} = \left\{ app(k,l,m) \in \mathcal{HB} \;\middle|\; \begin{array}{l}\text{if } m \text{ is a list}\\ \text{then } k \text{ and } l \text{ are lists}\\ \text{and } k\hat{\ }l = m\end{array} \right\}$$

for list splitting

$$S_{\mathrm{app}} = \left\{ app(k,l,m) \in \mathcal{HB} \;\middle|\; \begin{array}{l} k \text{ is a list,}\\ \text{if } l \text{ or } m \text{ is a list}\\ \text{then } l, m \text{ are lists}\\ \;\text{and } k\hat{\ }l = m\end{array} \right\}$$

more precise, for most usages

$$S_{\mathrm{app}} \subset S_{\mathrm{app},1} \cap S_{\mathrm{app},2}$$

Example (cont'd)

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for list appending

$$S_{\mathrm{app},2} = \left\{ app(k,l,m) \in \mathcal{HB} \;\middle|\; \begin{array}{l} \text{if } m \text{ is a list} \\ \text{then } k \text{ and } l \text{ are lists} \\ \text{and } k\hat{\ }l = m \end{array} \right\}$$

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$$S_{\mathrm{app}} = \left\{ app(k,l,m) \in \mathcal{HB} \;\middle|\; \begin{array}{l} k \text{ is a list,} \\ \text{if } l \text{ or } m \text{ is a list} \\ \text{then } l, m \text{ are lists} \\ \;\text{and } k\hat{\ }l = m \end{array} \right\}$$

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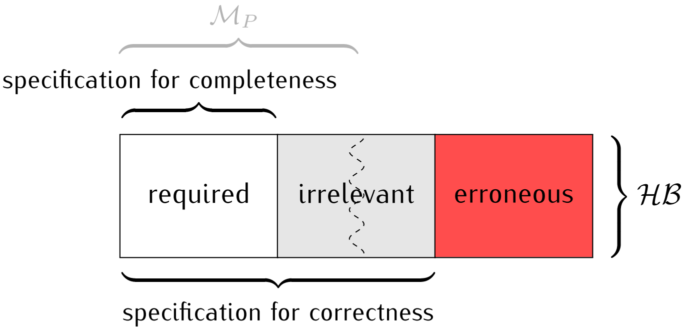
Approximate specifications

Usual situation:



$\mathcal{M}_P$

specification for completeness

required

irrelevant

erroneous

$\mathcal{HB}$

specification for correctness

Approximate specification: (S_{compl}, S_{corr})

Correctness$^+$: $S_{compl} \subseteq \mathcal{M}_P \subseteq S_{corr}$

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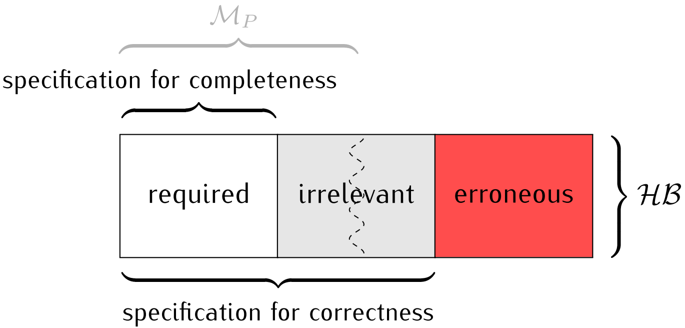

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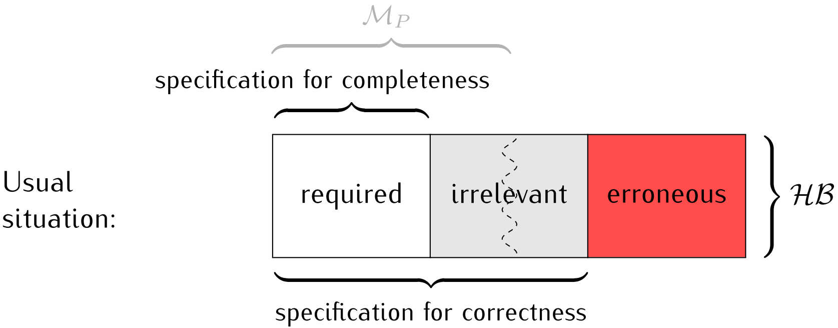
specification for correctness

Approximate specification: (S_{compl}, S_{corr})

Correctness$^+$: $S_{compl} \subseteq \mathcal{M}_P \subseteq S_{corr}$

When we build a program,
not known in advance if a given $A \in S_{compl} \setminus S_{corr}$ is in $\mathcal{M}_P$

Approximate specifications



Approximate specification: (S_{compl}, S_{corr})

Correctness$^+$: $S_{compl} \subseteq \mathcal{M}_P \subseteq S_{corr}$

$\mathcal{M}_P$ may differ in different programs for the same task
or at various stages of program development

Approximate specifications

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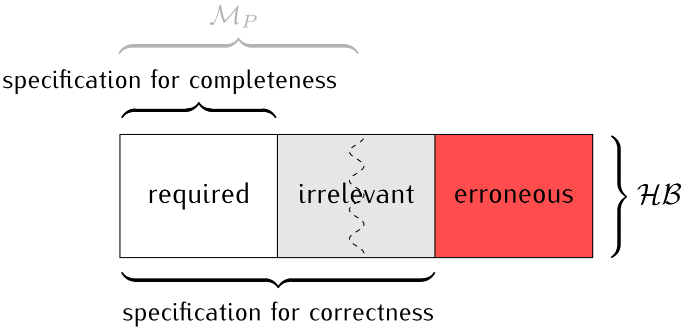

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Semantics preserving program transformations – too restrictive ☜

Example: [D_'18 TPLP]

Approximate specification, example, insertion sort

Ex. (we cannot know in advance, if $A \in \mathcal{M}_P$):

$insert/3$ – inserting a number into a sorted list

Should we accept $A = insert(2, [3, 1], [2, 3, 1])$? It's **irrelevant**!

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$$S_{insert} = \left\{ \begin{array}{r|l} insert(n, l_1, l_2) & n \notin \mathbb{Z}\text{, or} \\ \in \mathcal{HB} & l_1 \text{ not a sorted list} \\ & \text{of integers} \end{array} \right\} \cup S^0_{insert}$$

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$$S^0_{insert} = \left\{ \begin{array}{r|l} insert(n, l_1, l_2) & l_1, l_2 \text{ are sorted lists of integers,} \\ \in \mathcal{HB} & elms(l_2) = \{n\} \cup elms(l_1) \end{array} \right\}$$

where $elms(l)$ – the multiset of elements of l

$$S_{insert} = \left\{ \begin{array}{r|l} & \text{if } n \in \mathbb{Z} \text{ and} \\ insert(n, l_1, l_2) & l_1 \text{ is a sorted list of integers,} \\ \in \mathcal{HB} & \text{then } insert(n, l_1, l_2) \in S^0_{insert} \end{array} \right\}$$

Reasoning (declaratively) about correctness$^+$ of programs

- Proving correctness
- Proving completeness

Proving program correctness

Th. [Clark'79]: (the simplest theorem of LP $\ddot\smile$)

Let S – a specification, P – a program.

$$\boxed{\text{If } S \models P \text{ then } P \text{ correct w.r.t. } S.}$$

Proof: $S \models P \;\Rightarrow\; \mathcal{M}_P \subseteq S$ $\square$

Note: $S \models P$ means
for each ground instance $H \leftarrow B_1, \ldots, B_n$ of a clause of P
if $B_1, \ldots, B_n \in S$ then $H \in S$

The Th. – a declarative way to prove a declarative property.

The Th. should be well-known, but is unacknowledged.

Instead, more complicated methods based on operational semantics, on pre- and postconditions for LD-resolution [Bossi+Cocco'89,Apt'97,...].

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Example correctness proof

For each $H \leftarrow B_1, \ldots, B_n \in ground(P)$, if $B_1, \ldots, B_n \in S$ then $H \in S$.

Program + specification:

SPLIT:

$$s([\,],[\,],[\,]). \tag{1}$$

$$s([X|Xs],[X|Ys],Zs) \leftarrow s(Xs,Zs,Ys). \tag{2}$$

$$S = \{\, s(l,l_1,l_2) \mid l,l_1,l_2 \text{ are lists, } 0 \leq |l_1| - |l_2| \leq 1 \,\},$$

where $|l|$ – the length of a list l.

Proof:

Consider a ground instance $s([h|t],[h|t_2],t_1) \leftarrow s(t,t_1,t_2)$ of (2).
Assume $s(t,t_1,t_2) \in S$. Thus $[h|t],[h|t_2],t_1$ are lists. Let $m = |t_1| - |t_2|$.
As $m \in \{0,1\}$, we have $|[h|t_2]| - |t_1| = 1 - m \in \{0,1\}$.
So the head $s([h|t],[h|t_2],t_1)$ is in S. The proof for (1) is trivial.

Thus program SPLIT correct w.r.t. specification S.

Example correctness proof 2

$$\boxed{\text{If } S \models P \text{ then } P \text{ correct w.r.t. } S.}$$

We need to show:

for each $H \leftarrow B_1, \ldots, B_n \in ground(P)$ if $B_1, \ldots, B_n \in S$ then $H \in S$

$$S'_{\rm app} = \left\{ app(k,l,m) \in \mathcal{HB} \;\middle|\; \begin{array}{l} \text{if } l \text{ or } m \text{ is a list then} \\ k,l,m \text{ are lists and } k\hat{\ }l = m \end{array} \right\}$$

APP: $app(\, [\,], L, L\,).$ $\quad app(\, [H|K], L, [H|M]\,) \leftarrow app(\, K, L, M\,).$

Nontrivial part of a correctness proof for APP w.r.t. $S'_{\rm app}$:

Take a ground $\overbrace{app([h|k], l, [h|m])}^{H} \leftarrow \overbrace{app(k, l, m)}^{B}$, assume $B \in S'_{\rm app}$; assume l or $[h|m]$ is a list, show that $[h|k]\hat{\ }l = [h|m]$; so $H \in S_{\rm app}$.

(l or m is a list $\Rightarrow$ k, l, m are lists $\Rightarrow$ $k\hat{\ }l = m$)

Similar to informal reasoning about a program by a competent declarative programmer.

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Example correctness proof 3

MIDDLE: $middle(Mid, L) \leftarrow m(Mid, L, L).$ (1)

$m(E, [\_], [E|\_]).$ (2)

$m(E, [\_, \_|L1], [\_|L2]) \leftarrow m(E, L1, L2).$ (3)

$$\begin{aligned} S_{\mathrm{M}} = \; & \{\, middle(b_i, [b_1, \ldots, b_{2i-1}]) \in \mathcal{HB} \mid i > 0 \,\} \\ & \cup \{\, m(t, l, t') \in \mathcal{HB} \mid t' \text{ is not a list} \,\} \\ & \cup \{\, m(b_i, [a_1, \ldots, a_{2i-1}], [b_1, \ldots, b_n]) \in \mathcal{HB} \mid n \geq i > 0 \,\} \end{aligned}$$

If $S \models P$ then P correct w.r.t. S.

We need to show:

for each $H \leftarrow B_1, \ldots, B_n \in ground(P)$ if $B_1, \ldots, B_n \in S$ then $H \in S$.

Non-obvious part of a correctness proof for MIDDLE w.r.t. S_{M}

Take a ground instance $H \leftarrow B$ of (3). Show that if $B \in S_{\mathrm{M}}$ then $H \in S_{\mathrm{M}}$.

Example correctness proof 3 (cont'd)

$$m(E,[\_,\_|L1],[\_|L2]) \leftarrow m(E,L1,L2). \qquad (3)$$

$$\begin{aligned} S_{\rm M} = \ & \{\, middle(b_i,[b_1,\ldots,b_{2i-1}]) \in \mathcal{HB} \mid i>0 \,\} \\ & \cup \{\, m(t,l,t') \in \mathcal{HB} \mid t' \text{ is not a list} \,\} \\ & \cup \{\, m(b_i,[a_1,\ldots,a_{2i-1}],[b_1,\ldots,b_n]) \in \mathcal{HB} \mid n \geq i > 0 \,\} \end{aligned}$$

Take a ground instance $\overbrace{m(e,[e_1,e_2|l_1],[e_3|l_2])}^{H} \leftarrow \overbrace{m(e,l_1,l_2)}^{B}$ of (3).
Assume $B \in S_{\rm M}$. Then
1. l_2 is not a list, thus $H \in S_{\rm M}$, or
2. $e=b_i$, $l_1=[a_1,\ldots,a_{2i-1}]$, $l_2=[b_1,\ldots,b_n]$, $n \geq i > 0$. Hence
$H = m(b_i,[e_1,e_2,a_1,\ldots,a_{2i-1}],[e_3,b_1,\ldots,b_n])$. Renumber it:

$$H = m(b'_{i+1},[a'_1,\ldots,a'_{2(i+1)-1}],[b'_1,\ldots,b'_{n+1}]),$$

where $(n+1) \geq (i+1) > 0$.

Thus $H \in S_{\rm M}$. ☐

Similar to informal reasoning about a program by a competent declarative programmer.

Example correctness proof 3 (cont'd)

$$m(E, [\_, \_|L1], [\_|L2]) \leftarrow m(E, L1, L2). \qquad (3)$$

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Take a ground instance $\overbrace{m(e, [e_1, e_2|l_1], [e_3|l_2])}^{H} \leftarrow \overbrace{m(e, l_1, l_2)}^{B}$ of (3).
Assume $B \in S_{\mathrm{M}}$. Then

1. l_2 is not a list, thus $H \in S_{\mathrm{M}}$, or
2. $e = b_i$, $l_1 = [a_1, \ldots, a_{2i-1}]$, $l_2 = [b_1, \ldots, b_n]$, $n \geq i > 0$. Hence

$H = m(b_i, [e_1, e_2, a_1, \ldots, a_{2i-1}], [e_3, b_1, \ldots, b_n])$. Renumber it:

$$H = m(b'_{i+1}, [a'_1, \ldots, a'_{2(i+1)-1}], [b'_1, \ldots, b'_{n+1}]),$$

where $(n+1) \geq (i+1) > 0$.

Thus $H \in S_{\mathrm{M}}$. $\square$

Similar to informal reasoning about a program
by a competent declarative programmer.

Reasoning about program completeness

Surprising: the subject has been neglected ! $\ddot\frown$

Except for
[Deransart+Małuszyński'93], [Sterling+Shapiro'94] (informally),
[D_+Miłkowska'05], [D_'16,'18]; I am not aware of any other work.

[Hogger'84], [Kowalski'85] – the notion of completeness,
but not reasoning about it.

Semi-completeness

$$\boxed{\text{completeness} \;=\; \text{semi-completeness} \;+\; \text{termination}}$$

Df.: P is complete for a query Q w.r.t. S if for any ground $Q\theta$

$$S \models Q\theta \quad \Rightarrow \quad Q\theta \text{ is an answer for } P.$$

(P produces all the required answers for Q.)

Df.: P is semi-complete w.r.t. S if P is complete w.r.t. S for any query for which there exists a finite SLD-tree.

(P produces all the required answers, if the computation terminates.)

Lemma: If P is semi-complete w.r.t. S, and
P terminates (under some selection rule) for each query $A \in S$
then P is complete w.r.t. S.

Sufficient condition for completeness

$$\boxed{\text{completeness} \;=\; \text{semi-completeness} \;+\; \text{termination}}$$

Df.: $H \in \mathcal{HB}$ is covered by P w.r.t. S if there is a clause $(H \leftarrow A_1, \ldots, A_n) \in ground(P)$ in which $A_1, \ldots, A_n \in S$.

(A covered atom can be produced by a clause of P from atoms required by S to be produced.)

Th. (sufficient condition):
If each atom from S is covered w.r.t. S by P then P is semi-complete w.r.t. S.

Proving program termination – not discussed here.

Example

$S^0_{\rm app} = \{\, app(k,l,m) \in \mathcal{HB} \ \mid k,l,m$ are lists, $k\hat{\ }l = m \,\}$,

APP: $app(\,[\,],L,L\,).$
$app(\,[H|K],L,[H|M]\,) \leftarrow app(\,K,L,M\,).$

Let $H \in S^0_{\rm app}$. We show that H is covered by APP w.r.t. $S^0_{\rm app}$.

1. $H = app([\,],l,l)$. $H \in ground($APP$)$
2. $H = app(k,l,m)$, $k \neq [\,]$, $k\hat{\ }l = m$. So H is the head of $app([h|k'],l,[h|m']) \leftarrow app(k',l,m')$ and $app(k',l,m') \in S^0_{\rm app}$.

Thus APP semi-complete w.r.t. $S^0_{\rm app}$.

We know that P terminates for any query from S.

Hence APP complete w.r.t. $S^0_{\rm app}$.

Reasoning about completeness, comments

Not fully declarative, as termination is an operational property.

(Proving semi-completeness purely declarative)

But termination has to be established anyway.

So the not fully declarative approach seems reasonable.

A declarative sufficient condition exists [Deransart+Małuszyński'93].
But it leads to completeness proofs similar to
proving semi-completeness + termination.

Semi-completeness alone: Computation terminates ⇒
all required (by the specification) answers have been produced.

Proving correctness$^+$, comments

In my opinion

the sufficient conditions for correctness & (semi-) completeness

- are declarative – abstract from operational semantics
  (except for termination, which is needed anyway)
- are simple (cf. Hoare rules for imperative programming)
- correspond to a natural way of thinking by a declarative programmer
- can be used in every-day programming
  at various levels of (in)formality
- provide a guide how to reason about programs

Constructing correct$^+$ programs

How to construct a program

for an approximate specification $\mathcal{S} = (S_{compl}, S_{corr})$

Provide clauses so that

❶ each atom $A \in S_{compl}$ is covered (w.r.t. S_{compl}) by some clause

❷ each clause satisfies the sufficient condition for correctness (w.r.t. S_{corr})

(this produces a program correct and semi-complete w.r.t. $\mathcal{S}$;
not enough, $p(\vec{X}) \leftarrow p(\vec{X})$ possible)

❸ the clauses satisfy some sufficient condition for termination

informally: $p(\vec{s}) \leftarrow \ldots, p(\vec{t}), \ldots$
$\uparrow$ bigger terms $\uparrow$ smaller terms

Result: a program correct and complete [D_'18]

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Result: a program correct and complete [D_'18]

A more interesting example – file `ex.insert*.pdf`

Constructing correct$^+$ programs, example

Splitting a list into its odd- and even- numbered elements.

$$S = \{\, s(l, oe(l), ee(l)) \in \mathcal{HB} \mid l \text{ is a list} \,\}$$

where $oe(l)$ – the list of odd elements of list l

$(\, oe([e_1, \ldots, e_n]) = [e_1, e_3, ...] \,)$

$ee(l)$ – the list of even elements of list l

(e.g. $s([1,2,3,4,5],[1,3,5],[2,4]) \in S$)

An unusual case of exact specification!

Construct a program correct$^+$ w.r.t. (S, S).

Constructing correct$^+$ programs, example

Summary of the approach:
1. each atom $A \in S_{compl}$ is covered
2. each clause correct w.r.t. S_{corr}
3. termination…

$S = \{\, s(l, oe(l), ee(l)) \mid l \text{ is a list} \,\}$. Two kinds of elements of S:

1. $s([\,], [\,], [\,])$.
2. $A = s([h|t], [h|ee(t)], oe(t))$

Constructing correct$^+$ programs, example

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We need a $B \in S$ for clause body.

Preferably subterms of arguments of A should be used (for termination).

What about $[h|t] \leadsto t$?

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This suggests $A \leftarrow B \in ground(P)$

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We need a $B \in S$ for clause body.

Preferably subterms of arguments of A should be used (for termination).

What about $[h|t] \rightsquigarrow t$? $B = s(t, oe(t), ee(t))$?

This suggests $C_2 = s([H|T],[H|ET],OT) \leftarrow s(T,OT,ET)$.

(Each) A covered by C_2. $S \models C_2$.

P terminates for any query $s([e_1,\ldots,e_n],t,t')$ (maybe nonground).

$P = \{C_1, C_2\}$ correct & complete w.r.t. S.

Constructing correct$^+$ programs, comments

Summary of the approach:
1. each atom $A \in S_{compl}$ is covered
2. each clause correct w.r.t. S_{corr}
3. termination…

This proposal is rather obvious

I see it as

good practices of competent programmers made explicit

☞ Approximate specifications crucial

Beginning with exact specification $S_{compl} = S_{corr}$ is often unnecessary & counterproductive

We should not/cannot decide in advance
what *insert*/3 (of insertion sort) should do with unsorted lists
what *append*/3 should do with non-lists

On semantics-preserving program transformations

Program development: $P_1, \ldots, P_n$ $\quad \forall i \; S_{compl} \subseteq \mathcal{M}_{Pi} \subseteq S_{corr}$

The programs may be not equivalent

– distinct relations for the same predicate in P_i, P_j

$$\{\, q(\vec{t}\,) \in \mathcal{M}_{Pi} \,\} \neq \{\, q(\vec{t}\,) \in \mathcal{M}_{Pj} \,\}$$

The paradigm of
semantics-preserving program transformations

too restrictive

Ex.: Construction of SAT-solver [D_'18, Howe+King'12]

$P_1, P_2, P_3, \underset{\substack{\uparrow \\ \text{Prolog program}}}{P}$; distinct semantics of the main predicates in P_1, P_2.

[D_'18] illustrates the methods presented here + ...

More precisely

Program development: $P_1, \ldots, P_n \quad \forall i \; S_{compl,i} \subseteq \mathcal{M}_{Pi} \subseteq S_{corr,i}$

The specification is constant for some main predicates, so

$$\forall i \; S_{compl,q} \subseteq \{\, q(\vec{t}) \in \mathcal{M}_{Pi} \,\} \subseteq S_{corr,q}$$

Declarative diagnosis (algorithmic debugging)

All the declarativeness gone, when it comes to debugging

Next file of slides