On Domination and Control in Strategic Ability

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(joint work with Wojtek Jamroga and Damian Kurpiewski)

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Outline

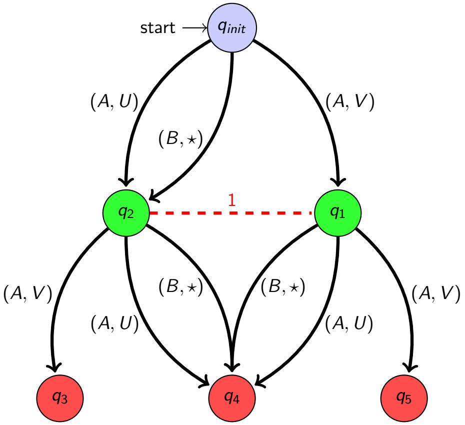
Enforceability in **CEGS**

- Models: Concurrent Epistemic Game Structures
- Verif. properties: Alternating-time Temporal Logic (**ATL**) - enforceability

$\langle\langle A \rangle\rangle \mathrm{F} \mathit{goal}$: coalition A has a collective strategy to enforce *goal* eventually.
I.e., $\exists_{\sigma_A} \langle \sigma_A \rangle \mathrm{F} \mathit{goal}$.

- Difficult: **NP**-complete and no fixed-point algorithms!
- Our contributions:
  - **new method of comparing partial strategies**
  - **application: alleviation of brute-force winning strategy synthesis**
  - application: strategy optimisation
  - a step towards distributed and parallel synthesis

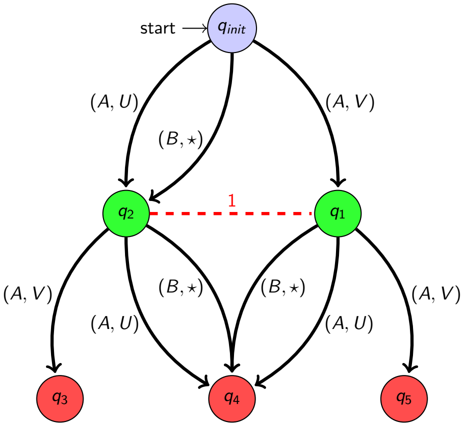
Concurrent Epistemic Game Structures



CEGS over $\mathbb{A}\text{gt} = \{1, \dots, k\}$ and *Act*:

- automaton with states *St*
- protocol $d_i(q) \subseteq Act$ for agent i
- transition function $o(q, \alpha_1, \dots, \alpha_k)$ *agent i selects action* α_i *in q...*
- labeling $V(p) \subseteq St$ of states with propositions
- $\sim_i$ indistinguishability relation for i

Concurrent Epistemic Game Structures - Strategies



Uniform Strategy for agent i:

- $\sigma_i : St \to Act$

  s.t. $q \sim_i q' \implies \sigma_i(q) = \sigma_i(q')$

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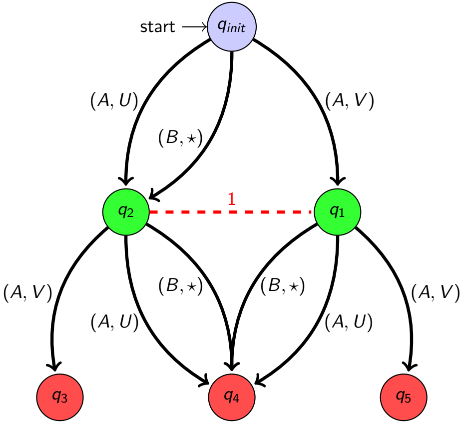

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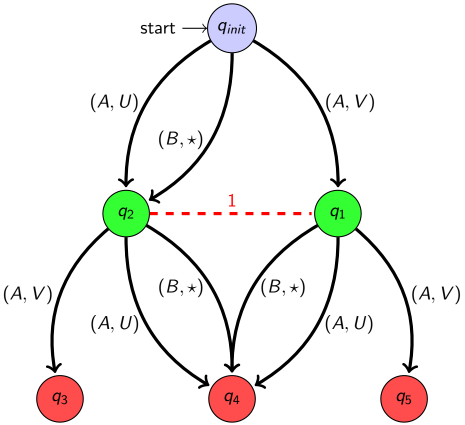

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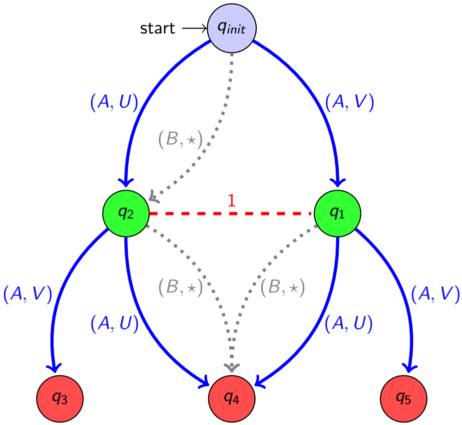

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Example:

- let $\sigma_1(q_{init}) = \sigma_1(q_1) = \sigma_1(q_2) = A$
- $\forall \pi \in out(q_{init}, \sigma_1)\ \exists_i \pi_i$ is red

  so: $q_{init} \models \langle \sigma_1 \rangle \mathrm{F} red$

Some Problems and Previous Results

- Checking $\langle\langle A \rangle\rangle \mathrm{F} \mathit{goal}$ under no memory and imperfect knowledge is **NP**-complete (and $\mathbf{\Delta_2^P}$-complete for whole $\mathbf{ATL}_{\mathrm{ir}}$).
- Standard fixed-point **ATL** equivalences don't work ...and $\mathbf{ATL}_{\mathrm{ir}}$ cannot be embedded in $\mathbf{AE}\mu\mathbf{C}$ $\Longrightarrow$ no fixed-point procedures? [1,2]
- (Symbolic) algorithms and methods mostly based on brute-force. [3,4,5]
- Exception: in [6] an approximate fixed-point verification put forward (but sometimes inconclusive).

1. Bulling, Jamroga: Alternating Epistemic Mu-Calculus, 2011.
2. Dima, Maubert, Pinchinat: Relating Paths in Transition Systems..., 2015.
3. Lomuscio, Raimondi: Automatic Verification of Multi-agent Systems by Model Checking..., 2007.
4. Busard et. al: Reasoning About Memoryless Strategies..., 2015.
5. Pilecki et. al: SMC: Synthesis of Uniform Strategies..., 2017.
6. Jamroga et. al: Approximate Verification of Strategic Abilities..., 2018.

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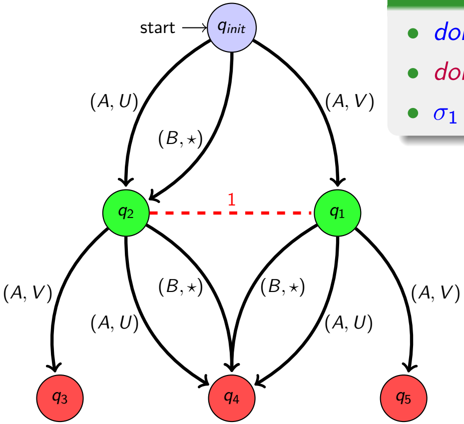
- Notions of outcome, etc. adapted to partial strategies.

Partial Strategies, ct'd



Example (part. strat. for 1):

- $dom(\sigma_1) = \{q_{init}\}$ with $\sigma_1(q_{init}) = A$
- $dom(\sigma'_1) = \{q_2\}$ with $\sigma'_1(q_2) = A$
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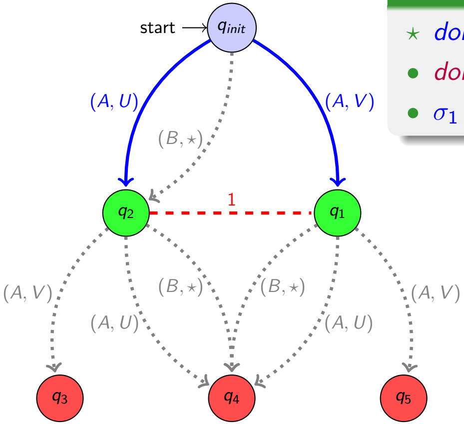

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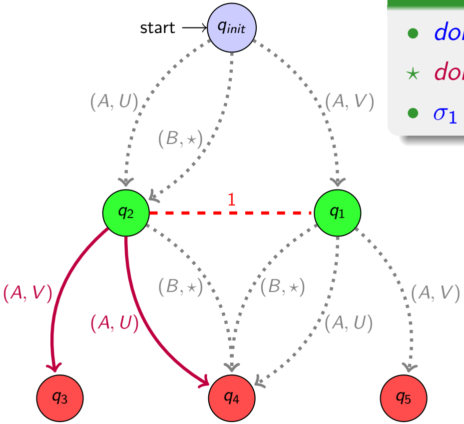

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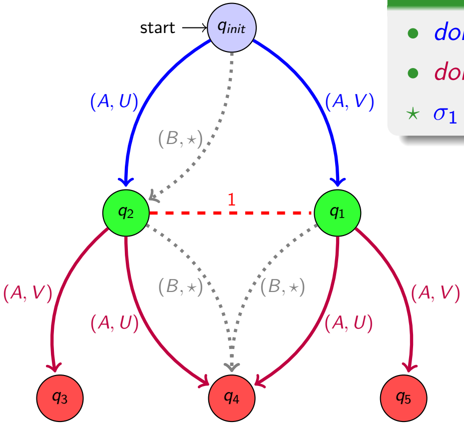

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Strategic domination

Let σ_A^C, σ_A and σ_A^C, σ'_A be conflictless. If $\mathcal{I}(\sigma_A^C, \sigma_A) = \mathcal{I}(\sigma_A^C, \sigma'_A)$ and for each $q \in \mathcal{I}(\sigma_A^C, \sigma_A)$ we have $\mathcal{IO}(\sigma_A^C, \sigma'_A)(q) \subseteq \mathcal{IO}(\sigma_A^C, \sigma_A)(q)$ then σ'_A dominates σ_A w.r.t. σ_A^C:

$$\sigma_A \preceq_{\sigma_A^C} \sigma'_A.$$

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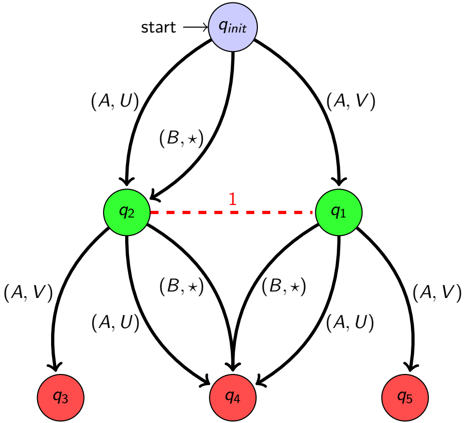

Example:

- $dom(\sigma_1^C) = \{q_{init}\}$ with $\sigma_1^C(q_{init}) = A$
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- $dom(\sigma'_1) = \{q_2\}$ with $\sigma'_1(q_2) = B$, $\mathcal{I}(\sigma_1^C, \sigma'_1) = \{q_1, q_2\}$, $\mathcal{IO}(\sigma_1^C, \sigma'_1)(q_2) = \{q_4\}$, $\mathcal{IO}(\sigma_1^C, \sigma'_1)(q_1) = \{q_4\}$.

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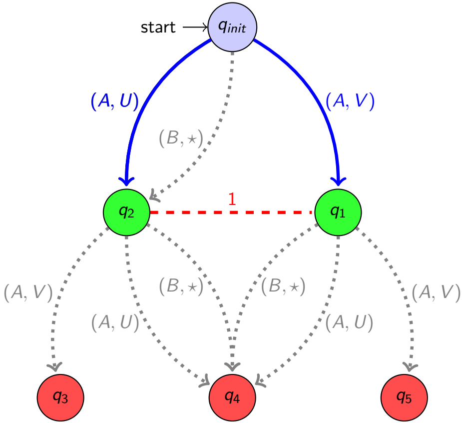

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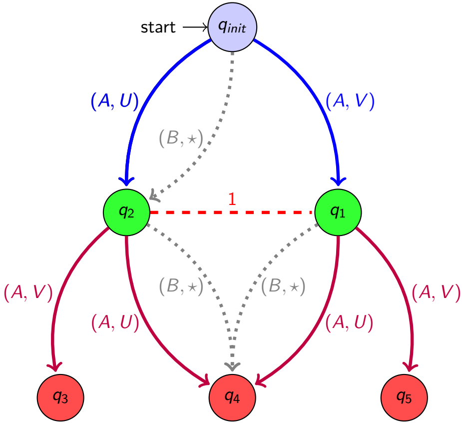

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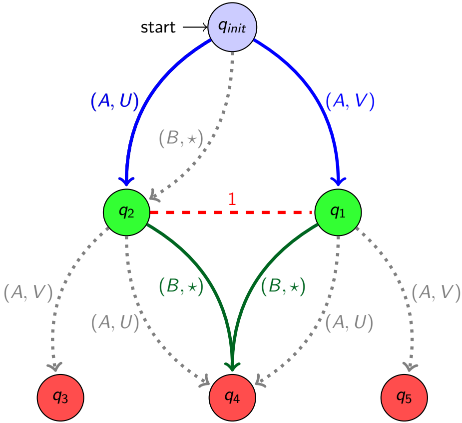

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Strategic Domination: Preserving Enforceability

Theorem

If:

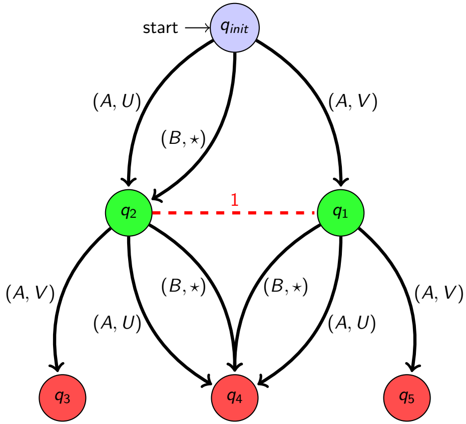
- σ_A^C, σ_A and σ_A^C, σ'_A conflictless
- $q_{init} \in dom(\sigma_A^C \cup \sigma_A)$ and $q_{init} \in dom(\sigma_A^C \cup \sigma'_A)$
- $\sigma_A \preceq_{\sigma_A^C} \sigma'_A$
- no q_{init}–loops

then:

$$q_{init} \models \langle \sigma_A^C \cup \sigma_A \rangle \mathrm{F} goal \ \ \textbf{implies} \ \ q_{init} \models \langle \sigma_A^C \cup \sigma'_A \rangle \mathrm{F} goal.$$

Deciding optimality of σ_A w.r.t. σ_A^C is co-**NP**-complete.

Switch Contexts for 'Global Optimisation'



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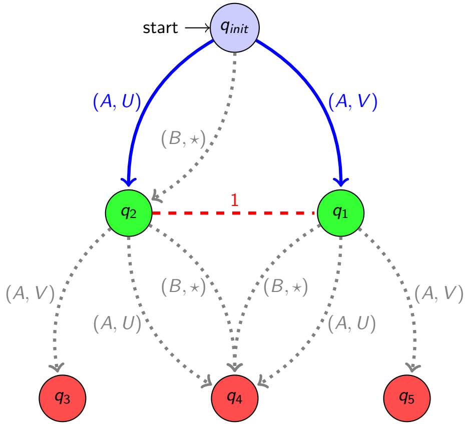

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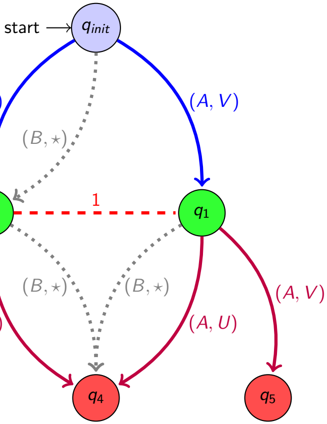

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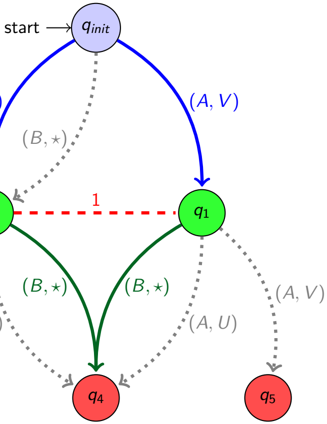

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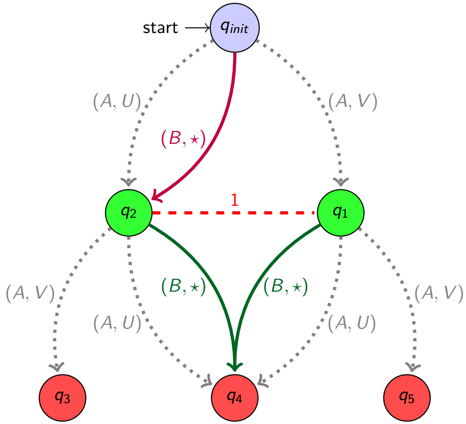

Model Checking via Simple(r) Strategy Synthesis

DominoDFS(σ_A)

Goal: find strategy σ'_A extending σ_A s.t. $q_{init} \models \langle\sigma'_A\rangle \mathrm{F} goal$.

1. IF $\langle\sigma_A\rangle \mathrm{F} goal$ THEN EXIT(σ_A)
2. CURR := a state reachable via σ_A s.t. CURR $\not\models goal$
3. CANDS := all non-dominated w.r.t. σ_A one-step strategies in CURR
4. FOREACH CAND in CANDS:
5. IF σ_A, CAND are conflictless AND $\sigma_A \cup$ CAND is loopless
   THEN CALL **DominoDFS($\sigma_A \cup$ CAND)**

Model Checking: (Simplified) Bridge Endings

Can N-S win?

K	DominoDFS	MCMAS	Approx.	Approx. opt.
1	0.0006	0.12	0.0008	< 0.0001
2	0.01	8712	0.01	< 0.0001
3	0.8	timeout	0.8	0.06
4	160	timeout	384	5.5
5	1373	timeout	8951	39
6	memout	timeout	memout	4524

K cards / hand, avg. of 50 random hand deals / experiment.

Model Checking: Castles

Can castles 1 and 2 defeat 3?

(c_1, c_2, c_3)	DominoDFS	MCMAS	SMC
$(1, 1, 1)$	0.3	65	63
$(2, 1, 1)$	1.5	12898	184
$(3, 1, 1)$	25	timeout	6731
$(2, 2, 1)$	25	timeout	4923
$(2, 2, 2)$	160	timeout	timeout
$(3, 2, 2)$	2688	timeout	timeout
$(3, 3, 2)$	timeout	timeout	timeout

Three castles, c_i workers / castle i; each worker can defend or attack; need to pause a round after defense.

(Approximate verifier can't handle it: inconclusive results.)

Conclusions

- Conceptual framework for pruning space of strategies.
- Applications in model checking (and strategy optimisation $\rightsquigarrow$ paper).
- When verifying enforceability under imperfect knowledge first try fixed-point approximations and then strategy pruning?
- *Future work:* full $\mathbf{ATL}_{\mathrm{ir}}$?
- *Future work:* parallelisation?

THANK YOU