Synthesis and analysis of concurrent processes in step semantics

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Joint work with Ryszard Janicki, Jetty Kleijn and Maciej Koutny

History

Structure of Concurrency.
R. Janicki, M. Koutny,
Theoretical Computer Science 112 (1):5-52 (1993)

- Set of **8 paradigms** was introduced
- **Weak composets** (combined posets) were shown as an **axiomatic model** for minimal signatures under **paradigm π_3**
- **No such solution** for the most general **paradigm π_1** was given

1993

History

Structure of Concurrency.

R. Janicki, M. Koutny,

Theoretical Computer Science 112 (1):5-52 (1993)

Weak composet is a triple $(X, \rightarrow, \nearrow)$, where

(WC1) $(X, \rightarrow)$ is an irreflexive poset,

$\nearrow$ is irreflexive

(WC2) $a \rightarrow b \;\Rightarrow\; a \nearrow b \wedge \neg(b \nearrow a)$

(WC3) $a \rightarrow b \nearrow c \vee a \nearrow b \rightarrow c \;\Rightarrow\; a \nearrow c$

(WC4) $a \rightarrow b \nearrow c \rightarrow d \;\Rightarrow\; a \rightarrow d$

(WC5) $a \nearrow b \rightarrow c \nearrow d \;\Rightarrow\; a \nearrow d \vee a = d$

1993

History

Semantics of Inhibitor Nets.

R. Janicki, M. Koutny,

Information and Computation 123:1-16 (1995)

Concurrent alphabets (Σ,sim,ser),

where sim is irreflexive and symmetric

and ser$\subseteq$sim

were introduced

1995

History

Semantics of Inhibitor Nets.
R. Janicki, M. Koutny,
Information and Computation 123:1-16 (1995)

Comtraces were defined,
as equivalence classes
of reflexive symmetric transitive closure
of relation ≈ on step sequences $wAz \approx wBCz$
where A,B,C are steps such that

$$B \cup C = A, \quad B \cap C = \emptyset \quad \text{and} \quad B \times C \subseteq ser$$

1995

Further simplification

A Characterization of Combined Traces Using Labeled Stratified Order Structures.
D.T.M. Lê,
ATPN’10:104-124 (2010)

Stratified order structure

is a triple $(X, \rightarrow, \nearrow)$, where

(S1) $\neg(a \nearrow a)$

(S2) $a \rightarrow b \;\Rightarrow\; a \nearrow b$

(S3) $a \nearrow b \nearrow c \wedge a \neq c \;\Rightarrow\; a \nearrow c$

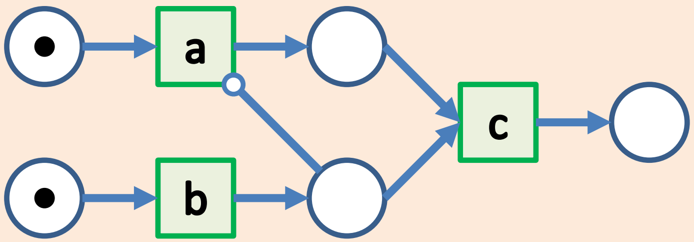
(S4) $a \rightarrow b \nearrow c \vee a \nearrow b \rightarrow c \;\Rightarrow\; a \rightarrow c$

2010

Complete picture

Paradigm π_3:

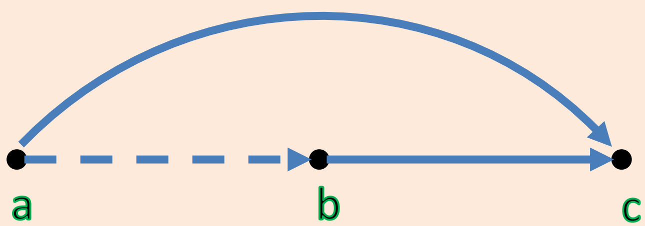
- 1-safe Inhibitor Petri Nets



- Combined Traces

[abc] = {abc, (ab)c}

- Weak Composets



Further development

Fundamentals of Modelling Concurrency Using Discrete Relational Structeres.
R. Janicki, M. Koutny,
Acta Informatica 34(5):367-388 (1997)

On Causality Semantics of Nets with Priorities.
R. Janicki, M. Koutny,
Fundamenta Informaticae 38(3):223-255 (1999)

Relational Structures Model of Concurrency.
R. Janicki,
Acta Informatica 45(4):279-320 (2008)

Modelling Concurrency with Comtraces and Generalized Comtraces.
R. Janicki, D.T.M. Lê,
Information and Computation 209(11):1355-1389 (2011)

Further development

Mutex Causality in Processes and Traces of General Elementary Nets.
J. Kleijn, M. Koutny,
Fundamenta Informaticae 122(1-2):119-146 (2013)

Algebraic Structure of Combined Traces.
Ł. Mikulski,
Logical Methods in Computer Science 9(3) (2013)

Lexicographical Generations of Combined Traces.
Ł. Mikulski, M. Piątkowski, S. Smyczyński,
ACSD'13:196-205 (2013)

Folded Hasse Diagrams of Combined Traces.
Ł. Mikulski, M. Koutny,
Information Processing Letters 114(4):208-216 (2014)

2013 2014

Solution

Characterising Concurrent Histories.
R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski
Fundamenta Informaticae 139 (1):21-42 (2015)

- **Separability** was introduced
- Generalized Mutex Order Structures (Δ,⊏,⇋) (later **Invariant Relational Structures**) were defined by **axioms** and **closure**
- **Szpilrajn Theorem** for GMOS has been formulated and proven

2015

Solution

Characterising Concurrent Histories.

R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski

Fundamenta Informaticae 139 (1):21-42 (2015)

Invariance axioms

(I1) $x \not\sqsubset x$

(I2) $x \neq y \wedge x \sqsubset z \sqsubset y \Rightarrow x \sqsubset y$

(I3) $x \rightleftharpoons y \Rightarrow y \rightleftharpoons x \neq y$

(I4) $x \prec y \sqsubset z \vee x \sqsubset y \prec z \Rightarrow x \rightleftharpoons z$

(I5) $z \rightleftharpoons y \wedge z \sqsubset x \sqsubset z \Rightarrow x \rightleftharpoons y$

(I6) $z \rightleftharpoons t \wedge x \sqsubset z \sqsubset y \wedge x \sqsubset t \sqsubset y \Rightarrow x \rightleftharpoons y$

2015

Solution

Characterising Concurrent Histories.
R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski
Fundamenta Informaticae 139 (1):21-42 (2015)

Closure

$(\Delta, \sqsubset, \rightleftharpoons) \mapsto (\Delta, \sqsubset^{\circledast} \circ (\rightleftharpoons \cup\, cross^{sym}) \circ \sqsubset^{\circledast}), \sqsubset^{\curlywedge})$

where

cross $= \{(x,y) \mid \exists_{z,t} : z \rightleftharpoons t \;\wedge\; x \sqsubset^{*} z \sqsubset^{*} y \;\wedge\; x \sqsubset^{*} t \sqsubset^{*} y\}$

Solution

Characterising Concurrent Histories.

R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski

Fundamenta Informaticae 139 (1):21-42 (2015)

Szpilrajn Theorem

For every GMO-structure *gmos*,

$$\text{os2los}(gmos) \neq \emptyset \text{ and } gmos = \bigcap \text{os2los}(gmos)$$

2015

Solution

Step Traces.
R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski
Acta Informatica 53 (1):35-65 (2016)

Concurrent alphabets (Σ,sim,seq),
where sim is irreflexive and symmetric
and seq\sim is symmetric
were introduced

2016

Solution

Step Traces.
R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski
Acta Informatica 53 (1):35-65 (2016)

Step Traces were defined,
as equivalence classes
of reflexive symmetric transitive closure
of relation ≈ on step sequences

$$wABz \approx wBAz \quad \text{or} \quad wCz \approx wDEz$$

where A,B,C,D,E are steps such that

$$A \cap B = \varnothing, \qquad A \times B \subseteq seq \cap seq^{-1}$$

$$D \cap E = \varnothing, \qquad D \cup E = C, \qquad \text{and} \qquad D \times E \subseteq seq \cap sim$$

2016

Solution

Step Traces.

R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski

Acta Informatica 53 (1):35-65 (2016)

Partition of Σ×Σ:

- ssi (strong simultaneity)
- sse (semi-sequentializability)
- con (concurrency)
- wdp (weak dependence)
- rig (rigid order)
- inl (interleaving)

2016

Examples

Suppose that we can **execute** two basic instructions *a* and *b* in **three modes**:

- *a* before *b*
- *b* before *a*
- *a* together with *b* (in first step we read all the utilized values, then we write results)

When **results** will be **equal**?

Examples

x = 5; y = 7;

a: x = x+2;
b: y = y+3;

a: x = 2x;
b: x = x+4;

a: x = x+2;
b: x = x+3;

a: x = y;
b: y = x;

a: x = x+2;
b: y = x+3;

a: x = y+2;
b: y = y+3;

Examples

x = 5; y = 7;

a: x = x+2;
b: y = y+3;

[(7,10),(7,10),(7,10)]

a: x = 2x;
b: x = x+4;

[(14),(18),(9 or 10)]

a: x = x+2;
b: x = x+3;

[(10),(10),(7 or 8)]

a: x = y;
b: y = x;

[(7,7),(5,5),(7,5)]

a: x = x+2;
b: y = x+3;

[(7,10),(7,8),(7,8)]

a: x = y+2;
b: y = y+3;

[(9,10),(12,10),(9,10)]

Examples

x = 5; y = 7;

concurrency

a: x = x+2;

b: y = y+3;

[(7,10),(7,10),(7,10)]

rigid order

a: x = 2x;

b: x = x+4;

[(14),(18),(9 or 10)]

interleaving

a: x = x+2;

b: x = x+3;

[(10),(10),(7 or 8)]

strong simultaneity

a: x = y;

b: y = x;

[(7,7),(5,5),(7,5)]

weak dependence

a: x = x+2;

b: y = x+3;

[(7,10),(7,8),(7,8)]

semi-sequentializability

a: x = y+2;

b: y = y+3;

[(9,10),(12,10),(9,10)]

Solution

Step Traces.
R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski
Acta Informatica 53 (1):35-65 (2016)

(Saturated) Structure of a step sequence:

Let u be a step sequence over $(\Sigma,\text{sim},\text{seq})$.

$$u \rightarrow (\Delta,\sqsubset,\rightleftharpoons,l), \text{ where } \Delta=\text{occ}(u),$$

and for all $\alpha,\beta\in\text{occ}(u)$ with $\text{pos}_u(\alpha)=k$, $\text{pos}_u(\beta)=m$

$$\alpha \rightleftharpoons \beta \quad \text{if } k\neq m$$

$$\alpha \sqsubset \beta \quad \text{if } k\leq m \wedge \alpha\neq\beta$$

2016

Solution

Step Traces.

R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski

Acta Informatica 53 (1):35-65 (2016)

(Dependence) Structure of a step sequence:

Let *u* be a step sequence over (Σ,sim,seq).

$u \rightarrow (\Delta, \sqsubset, \rightleftharpoons, l)$, where $\Delta = occ(u)$,

and for all $\alpha,\beta \in occ(u)$ with $pos_u(\alpha)=k$, $pos_u(\beta)=m$

$\alpha \rightleftharpoons \beta$ if $(l(\alpha),l(\beta)) \notin sim \cap seq \wedge k<m$

or $(l(\alpha),l(\beta)) \notin sim \cap seq^{-1} \wedge k>m$

$\alpha \sqsubset \beta$ if $(l(\alpha),l(\beta)) \notin seq \cap seq^{-1} \wedge k<m$

or $(l(\alpha),l(\beta)) \in sim \setminus seq^{-1} \wedge k=m$

2016

Solution

Step Traces.

R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski

Acta Informatica 53 (1):35-65 (2016)

(Invariant) Structure of a step sequence:

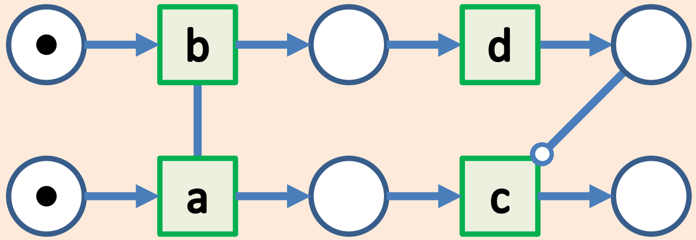
- additional axiom – label linearity:
  (I7) $x=a^{(i)} \wedge y=a^{(j)} \wedge i<j \Rightarrow x \prec y$
- closure as in the unlabelled case

2016

Complete picture

Paradigm π_1:

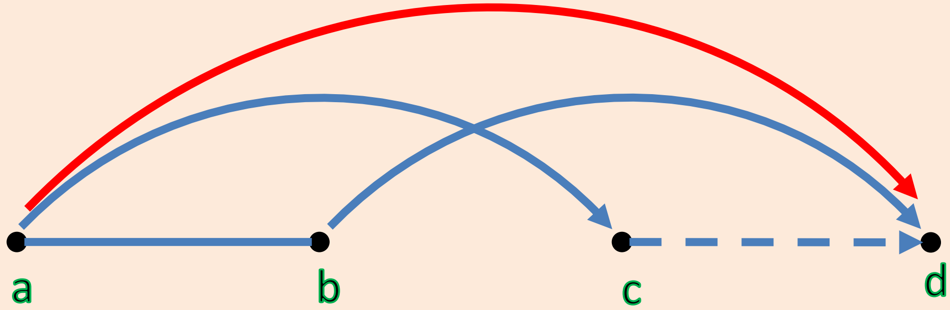
- 1-safe Inhibitor Petri Nets with Mutexes



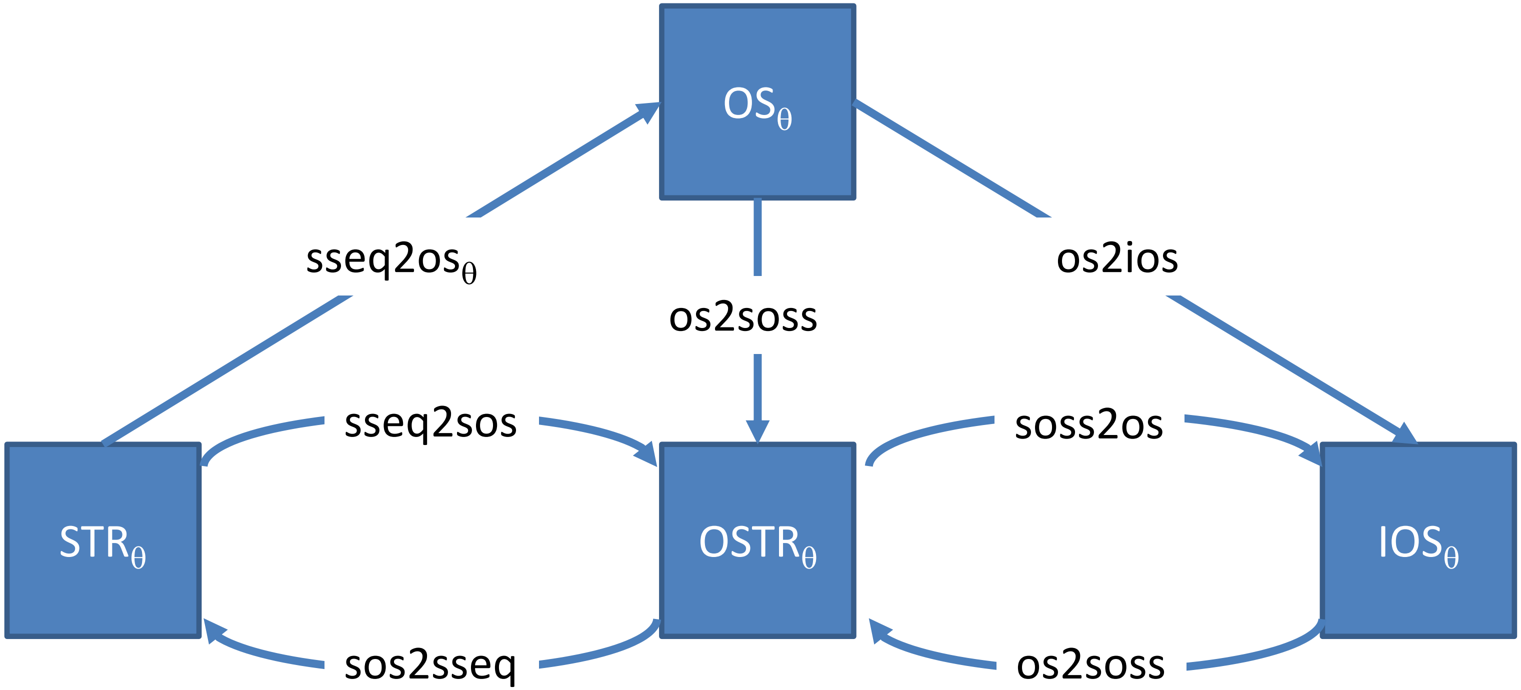
- Step Traces

[abcd] = {abcd, ab(cd), bacd, ba(cd), acbd, a(bc)d}

- Invariant Structures



Diagram



Theorem: The above diagram commutes.

Synthesis

Invariant Structures and Dependence Relations.

R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski

Fundamenta Informaticae 155 (1-2):1-29 (2017)

Evidence relations:

$\longrightarrow_{or} = \{(x,y) \mid x \sqsubset y \not\sqsubset x \rightleftharpoons y\}$

$\longleftarrow_{or} = \{(x,y) \mid x \not\sqsubset y \sqsubset x \rightleftharpoons y\}$

$\dashrightarrow_{or} = \{(x,y) \mid x \sqsubset y \not\sqsubset x \not\rightleftharpoons y\}$

$\dashleftarrow_{or} = \{(x,y) \mid x \not\sqsubset y \sqsubset x \not\rightleftharpoons y\}$

$—_{or} = \{(x,y) \mid x \not\sqsubset y \not\sqsubset x \rightleftharpoons y\}$

$\leftarrow\!\!\rightarrow_{or} = \{(x,y) \mid x \sqsubset y \sqsubset x \not\rightleftharpoons y\}$

$---_{or} = \{(x,y) \mid x \not\sqsubset y \not\sqsubset x \not\rightleftharpoons y \neq x\}$

2017

Synthesis

Invariant Structures and Dependence Relations.

R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski

Fundamenta Informaticae 155 (1-2):1-29 (2017)

Evidences of the dependence structures:

	rig	inl	ssi	sse	wdp	con
$pos_u(x)<pos_u(y)$	→	—	→	- -→	→	- - -
$pos_u(x)=pos_u(y)$			←-→	- -→	←- -	- - -
$pos_u(x)>pos_u(y)$	←	—	←	←	←- -	- - -

2017

Synthesis

Invariant Structures and Dependence Relations.

R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski

Fundamenta Informaticae 155 (1-2):1-29 (2017)

Observable (possible) evidences:

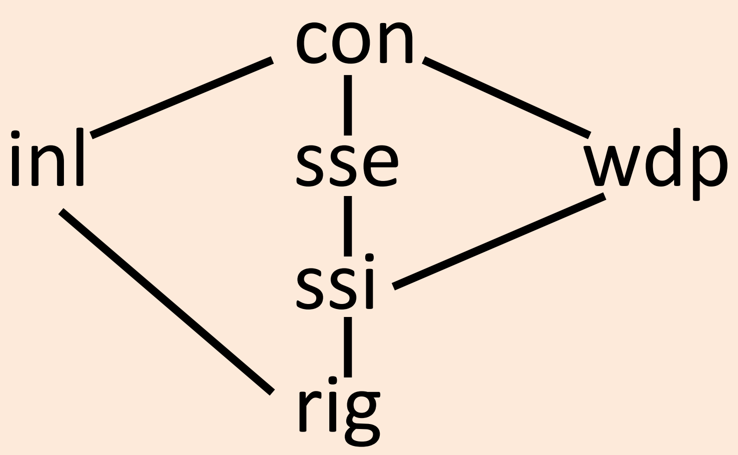
	core	implied
rig	← →	
inl	—	← →
ssi	⇠⇢ ← →	
sse	⇢ ←	⇠⇢ →
wdp	⇠ →	⇠⇢ ←
con	---	— ⇢ ⇠ ⇠⇢ ← →

2017

Synthesis

Invariant Structures and Dependence Relations.
R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski
Fundamenta Informaticae 155 (1-2):1-29 (2017)

Lattice of evidences (Hasse diagram):



Synthesis

Invariant Structures and Dependence Relations.
R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski
Fundamenta Informaticae 155 (1-2):1-29 (2017)

Important notions:

- consistence with alphabet
- evidence consistence with alphabet
- core reduction
- canonical step alphabet

2017

Synthesis

Invariant Structures and Dependence Relations.
R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski
Fundamenta Informaticae 155 (1-2):1-29 (2017)

Theorem

Invariant relational structure *ir*
is consitent with a step alphabet over Σ

if and only if

ir is consistent with its canonical step alphabet

2017

Synthesis

Alphabet of Acyclic Invariant Structures.

R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski

Fundamenta Informaticae 154 (1-4):207-224 (2017)

Structural restriction:

- weak causality relation of an invariant structure is acyclic

Consequences:

- axiomatization and closure are simpler
- reduction of a structure is possible

2017

Analysis

Classifying Invariant Structures of Step Traces.

R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski

Journal of Computer and System Sciences 104: 297-322 (2019)

Alphabet restriction:

- particular relations based on sim and seq are empty

2019

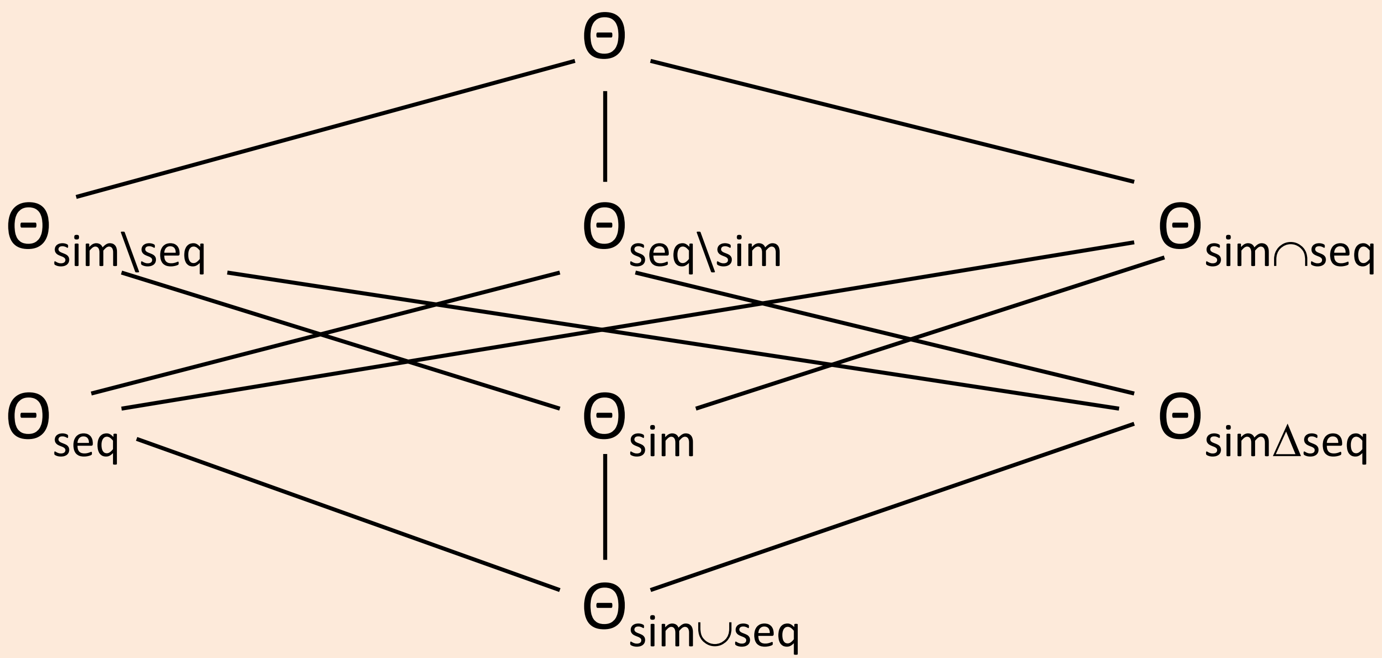
Analysis

Classifying Invariant Structures of Step Traces.

R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski

Journal of Computer and System Sciences 104: 297-322 (2019)

Inclusion diagram:



Analysis

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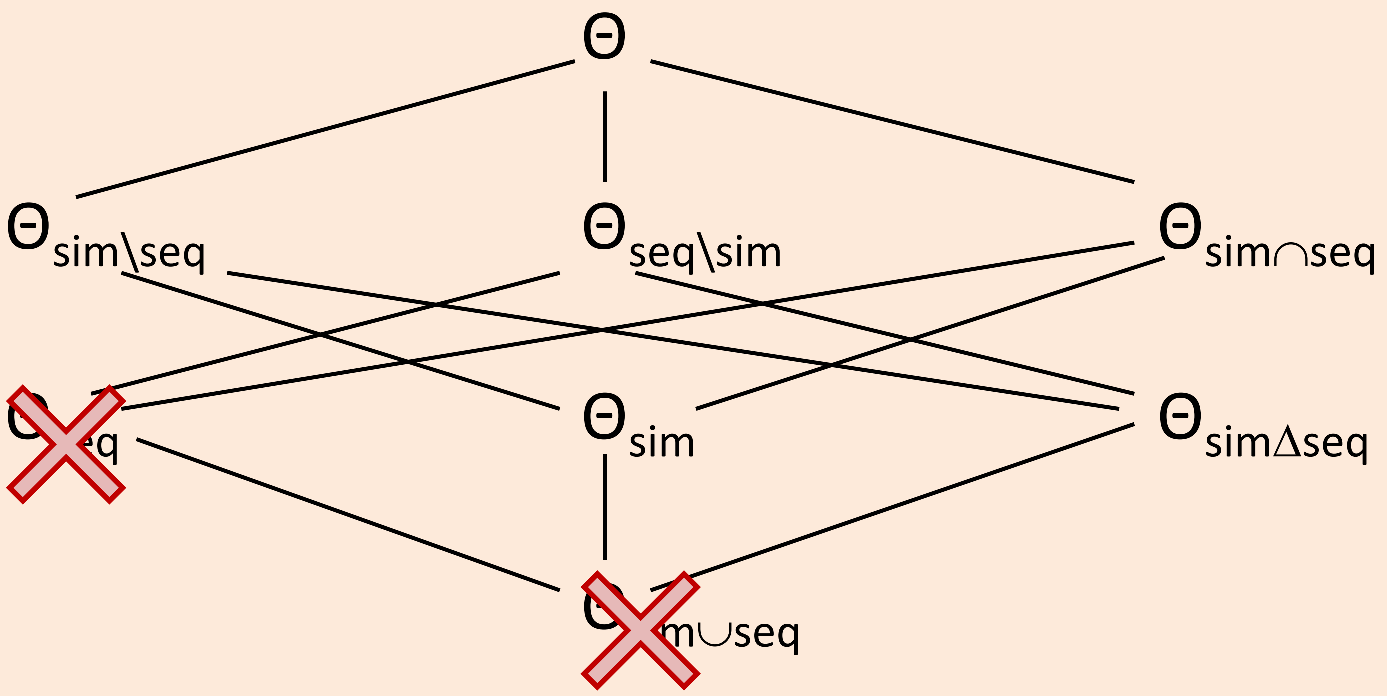

2019

Analysis

Classifying Invariant Structures of Step Traces.

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Journal of Computer and System Sciences 104: 297-322 (2019)

Discussed subclasses (5 interesting ones):

- Θ_{sim} – Mazurkiewicz traces (sequential)
- $\Theta_{sim \setminus seq}$ – satisfying the paradigm π_2
- $\Theta_{sim \cap seq}$ – completely new model
- $\Theta_{seq \setminus sim}$ – Comtraces (paradigm π_3)
- $\Theta_{sim \Delta seq}$ – Mazurkiewicz tr. (steps, paradigm π_8)

2019

Analysis

Classifying Invariant Structures of Step Traces.

R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski

Journal of Computer and System Sciences 104: 297-322 (2019)

Important properties (for every class):

- defining property at the level of relational structures is proposed
- more restricitive axiomatisation of invariant strucutures is proposed
- axiomatizations are completely consistent with defining properties

2019

More precise analysis

A Precise Characterisation of Step Traces and Their Concurrent Histories.
R. Janicki, J. Kleijn, Ł. Mikulski
Scientific Annals of Computer Science 28(2): 237-267 (2018)

Alphabet restriction:

- particular relations based on ssi, sse/wdp, con, inl emptiness
- only rig is always assumed to be nonempty

2018

More precise analysis

A Precise Characterisation of Step Traces and Their Concurrent Histories.

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Scientific Annals of Computer Science 28(2): 237-267 (2018)

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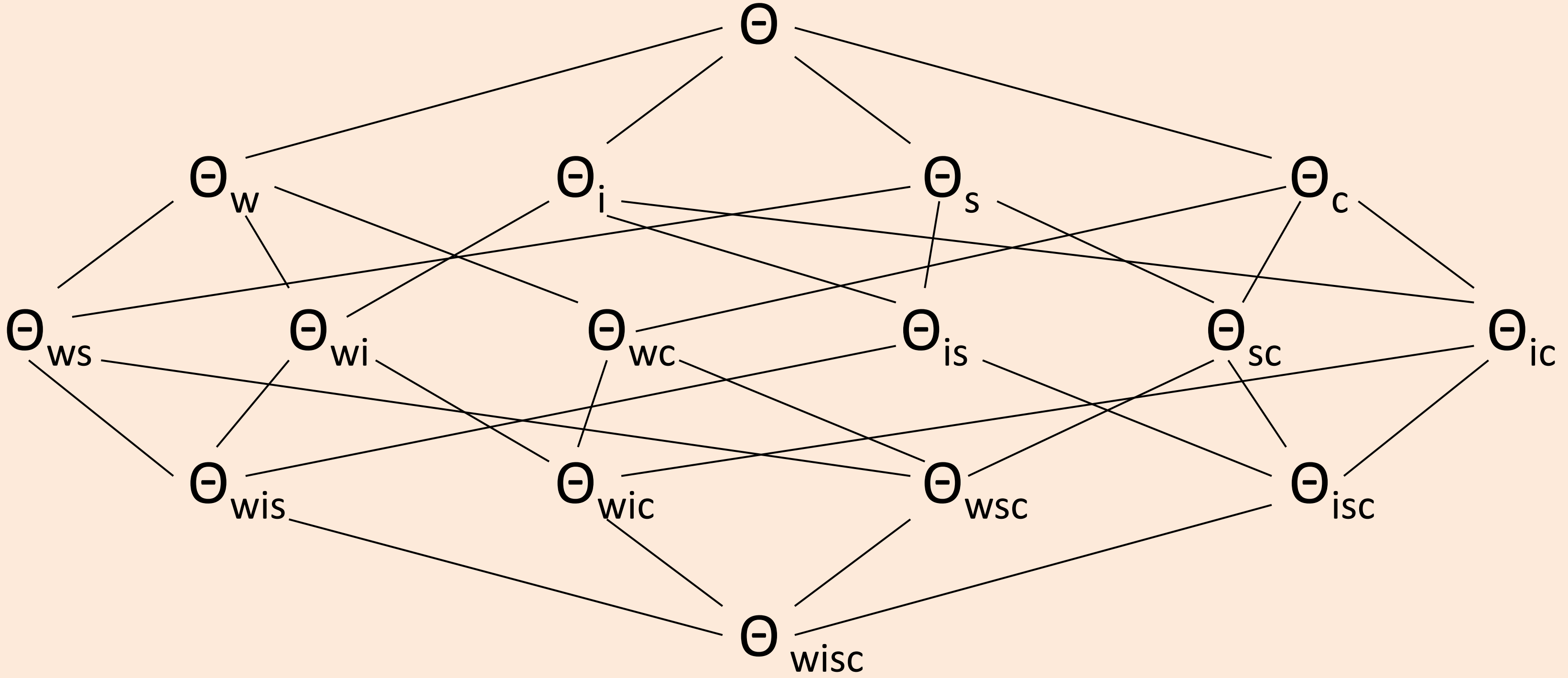

2018

More precise analysis

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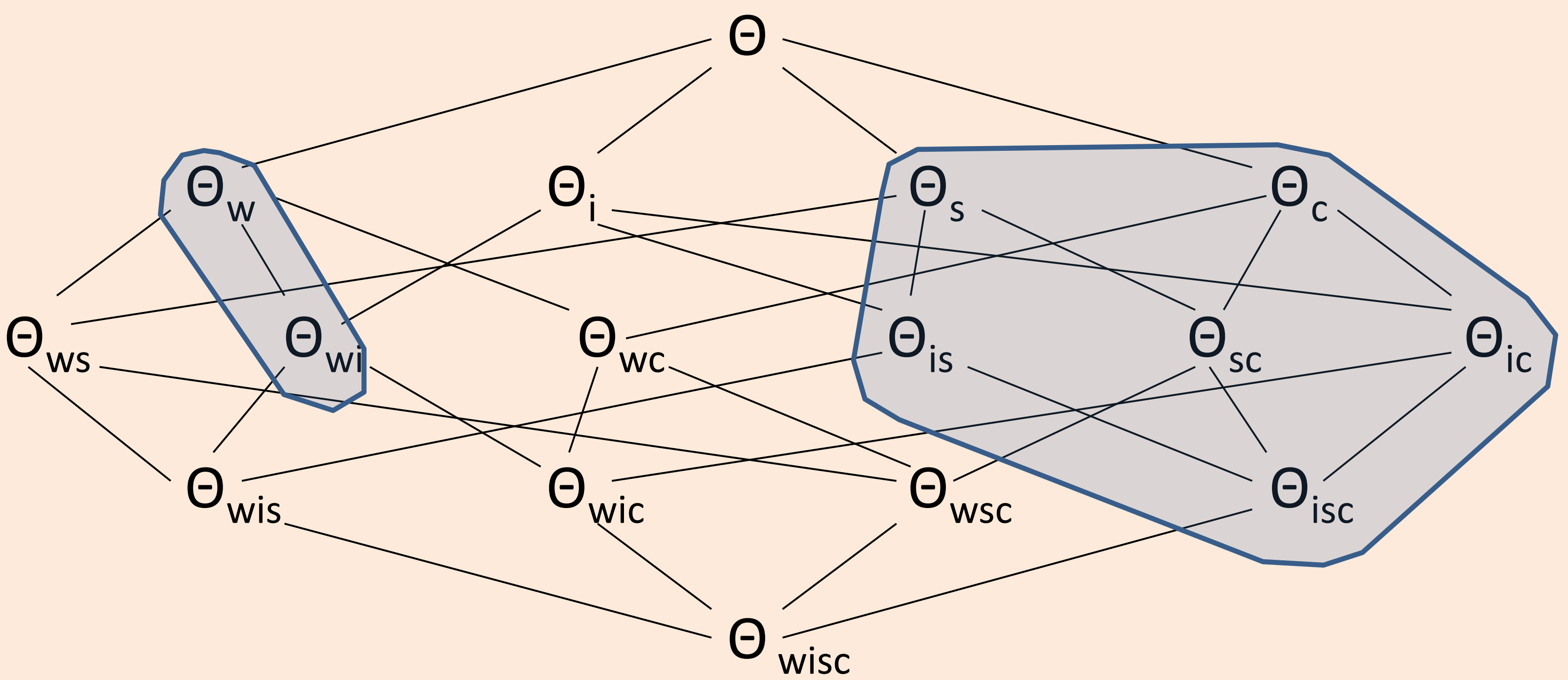
Scientific Annals of Computer Science 28(2): 237-267 (2018)

New classes:



2018

More precise analysis

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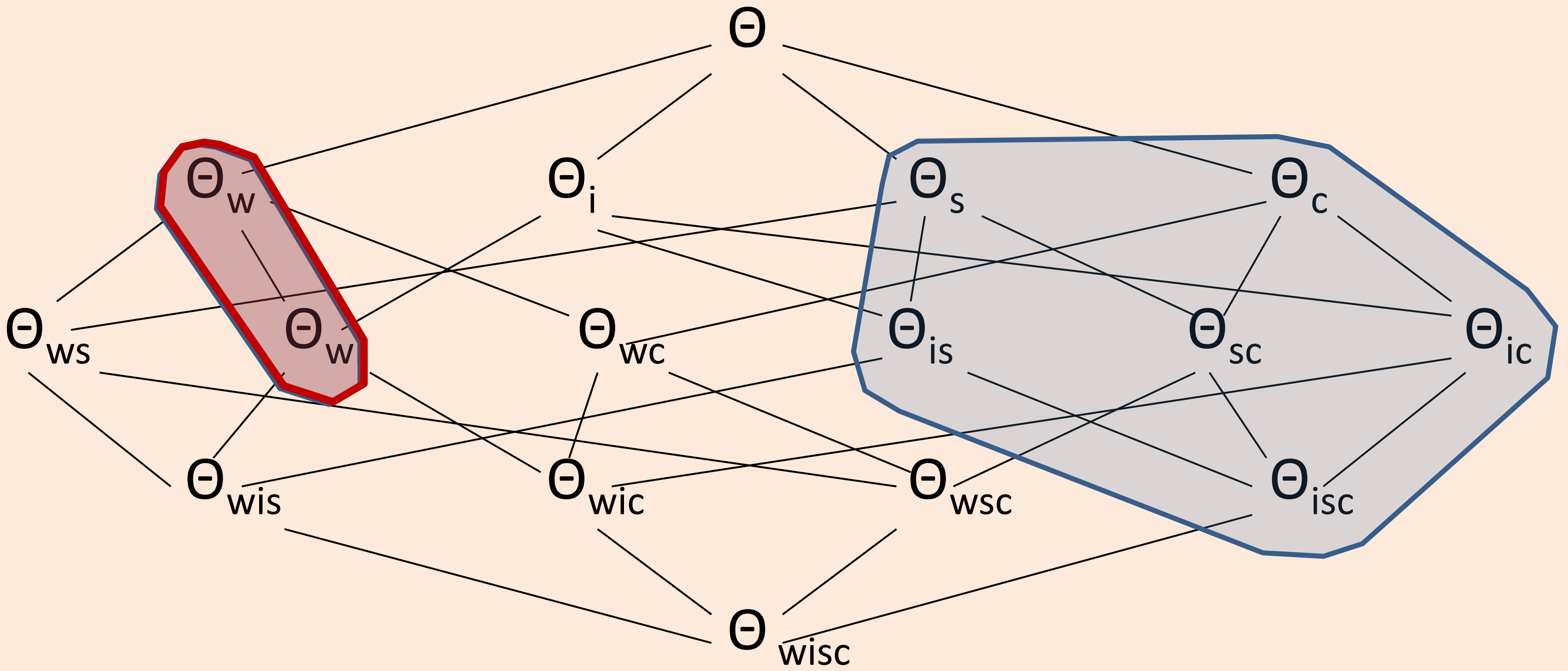
Scientific Annals of Computer Science 28(2): 237-267 (2018)

Alphabets without weak dependence:



2018

More precise analysis

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Scientific Annals of Computer Science 28(2): 237-267 (2018)

Important properties (for every class):

- axiomatizations are completely consistent with defining properties
- no fake invariant structures
- very regular

2018

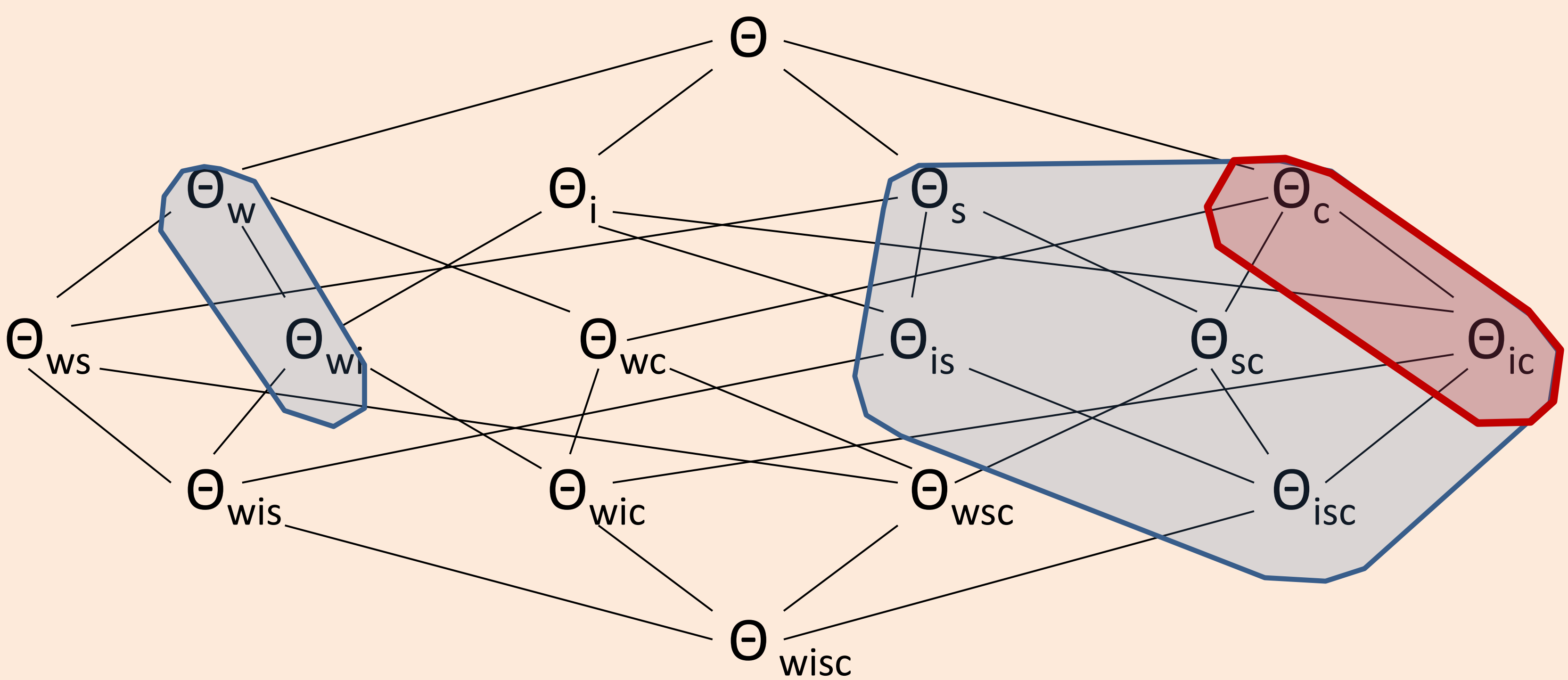
More precise analysis

A Precise Characterisation of Step Traces and Their Concurrent Histories.

R. Janicki, J. Kleijn, Ł. Mikulski

Scientific Annals of Computer Science 28(2): 237-267 (2018)

Aphabets without true concurrency:



More precise analysis

A Precise Characterisation of Step Traces and Their Concurrent Histories.
R. Janicki, J. Kleijn, Ł. Mikulski
Scientific Annals of Computer Science 28(2): 237-267 (2018)

Important properties (for every class):

- axiomatizations are completely consistent with defining properties
- fake invariant structures exist (invariant structures from other groups satisfy defining property)

2018

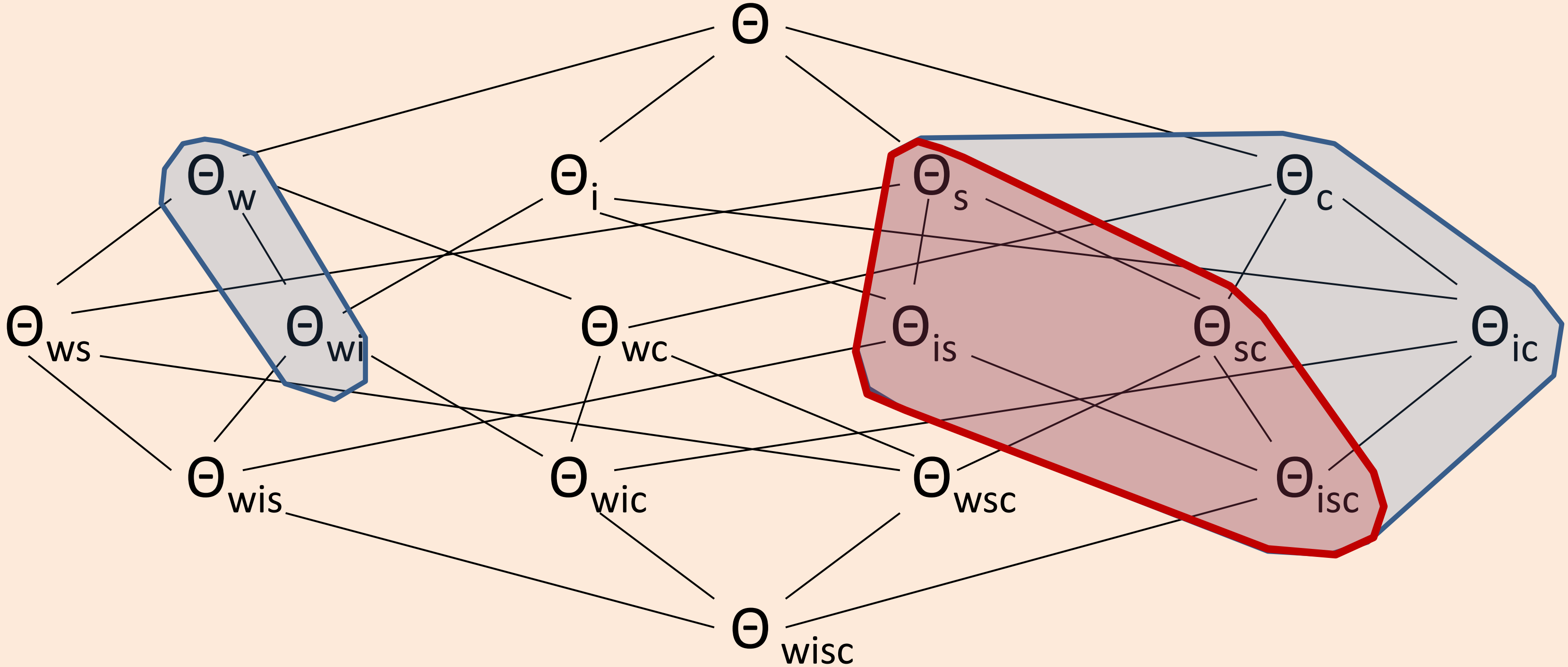
More precise analysis

A Precise Characterisation of Step Traces and Their Concurrent Histories.

R. Janicki, J. Kleijn, Ł. Mikulski

Scientific Annals of Computer Science 28(2): 237-267 (2018)

Alpgabets without strong simultaneity:



More precise analysis

A Precise Characterisation of Step Traces and Their Concurrent Histories.
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Scientific Annals of Computer Science 28(2): 237-267 (2018)

Important properties (for every class):

- there is an inconsistency between axiomatizations and defining properties
- fake invariant structures exist
- very irregular

2018

Algebraic Properties

Algebraic Structure of Step Traces and Interval Traces.

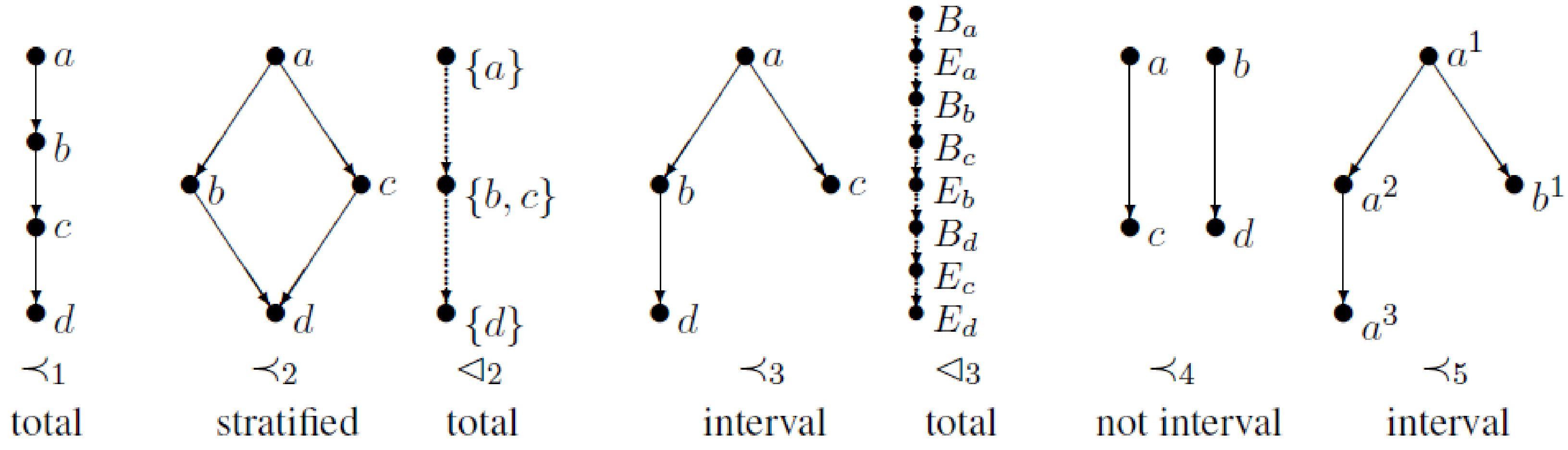
R. Janicki, Ł. Mikulski

Fundamenta Informaticae 175(1-4): 253-280 (2020)

Interval orders:

A partial order $< \subseteq X \times X$ is **interval** if

$$\forall_{a,b,c,d \in X} \ (a<c \wedge b<d) \Rightarrow a<d \vee b<c$$



Algebraic Properties

Algebraic Structure of Step Traces and Interval Traces.

R. Janicki, Ł. Mikulski

Fundamenta Informaticae 175(1-4): 253-280 (2020)

Projection representation:

sequences

$$\Pi_\Delta(ax) = \begin{cases} a\Pi_\Delta(x) & \text{if } a \in \Delta \\ \Pi_\Delta(x) & \text{if } a \notin \Delta \end{cases}$$

step sequences

$$\Pi_{a,b}(A) = \Pi_{b,a}(A) = \begin{cases} \lambda & \text{if} \quad \{a,b\} \cap A = \varnothing \\ a & \text{if} \quad \{a,b\} \cap A = \{a\} \\ b & \text{if} \quad \{a,b\} \cap A = \{b\} \\ ba & \text{if} \quad \{a,b\} \subseteq A \wedge (a,b) \in wdp \\ ab & \text{if} \quad \{a,b\} \subseteq A \wedge (a,b) \in sse \\ \bot & \text{if} \quad \{a,b\} \subseteq A \wedge (a,b) \in ssi \end{cases}$$

interval sequences

$$\dot{\Pi}_{a,b}(x) = \Pi_{\{B_a,E_a,B_b,E_b\}}(x).$$

2020

Algebraic Properties

Algebraic Structure of Step Traces and Interval Traces.

R. Janicki, Ł. Mikulski

Fundamenta Informaticae 175(1-4): 253-280 (2020)

Theorem

Two sequences are equivalent

iff

they have equal projection representations.

Algebraic Properties

Algebraic Structure of Step Traces and Interval Traces.
R. Janicki, Ł. Mikulski
Fundamenta Informaticae 175(1-4): 253-280 (2020)

Hiding representation:

sequences

$$\Xi_a(bx) = \begin{cases} a\ \Xi_a(x) & \text{if } b = a \\ \bot\ \Xi_a(x) & \text{if } b \neq a \wedge (a,b) \in dep \\ \Xi_a(x) & \text{otherwise} \end{cases}$$

interval traces

$$\dot{\Xi}_a(\alpha x) = \begin{cases} \alpha\ \dot{\Xi}_a(x) & \text{if } \alpha = B_a \vee \alpha = E_a \\ \bot\ \dot{\Xi}_a(x) & \text{if } \alpha \neq B_a \wedge \alpha \neq E_a \wedge ((\alpha, E_b) \in dep \vee (\alpha, B_b) \in dep) \\ \dot{\Xi}_a(x) & \text{if } \alpha \neq B_a \wedge \alpha \neq E_a \wedge ((\alpha, E_b) \in ind \vee (\alpha, B_b) \in ind) \end{cases}$$

Algebraic Properties

Algebraic Structure of Step Traces and Interval Traces.

R. Janicki, Ł. Mikulski

Fundamenta Informaticae 175(1-4): 253-280 (2020)

Hiding representation:

step sequences

$$\Xi_a(A) = \begin{cases} \bot^{|\vdash_a^A|}\, a\, \propto^{|\bowtie_a^A|} \bot^{|\dashv_a^A|} & \text{if } a \in A \\ \bot^{|\top_a^A|} & \text{otherwise} \end{cases}$$

where

$$\begin{aligned} \bowtie_a^A &= \{b \in A \setminus \{a\} &&\mid (a,b) \in ((ssi \cup wdp)|_A)^{\circledast}\} \\ \vdash_a^A &= \{b \in A \setminus \bowtie_a^A &&\mid (a,b) \in wdp\} \\ \dashv_a^A &= \{b \in A \setminus \bowtie_a^A &&\mid (a,b) \in sse\} \\ \top_a^A &= \{b \in A &&\mid (a,b) \notin con \cup inl\} \end{aligned}$$

Algebraic Properties

Algebraic Structure of Step Traces and Interval Traces.
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Theorem

Two sequences are equivalent

iff

they have equal hiding representations.

Algebraic Properties

Algebraic Structure of Step Traces and Interval Traces.

R. Janicki, Ł. Mikulski

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Extremal representations:

- minimal lexicographical canonical form
- maximal lexicographical canonical form
- greedy maximal concurrent form
- greedy minimal concurrent form
- maximally concurrent form
- minimally concurrent form
- Foata normal form (only for interval traces)

2020

Abstract View

Relational Structures for Concurrent Behaviours.

R. Janicki, M. Koutny, J. Kleijn, Ł. Mikulski

Theoretical Computer Science (2020, in Press)

Central message:

*After specifying **the set R of relational structures** which represent an application specific class of concurrent behaviours, the development of **a complete framework** is basically **automatic.***

2020

Thank you!