

Ride Sharing Platform Vs Taxi Platform: the Impact on the Revenue

Benjamin Bordais¹,
Costas Courcoubetis²

¹ENS Rennes

²Singapore University of Technology and Design

March 28, 2019

There is great need of mobility in major cities!

Possibilities to get a ride:

- Public transportation
- One's own car
- Ride sharing
- A taxi



Consists in optimizing the choices of a platform

- Choose efficiently the point of departure and arrival¹
- An optimization algorithm to efficiently match supply and demand²

¹ Service region design for urban electric vehicle sharing systems, Long He et al., 2017

² On-demand high-capacity ride-sharing via dynamic trip-vehicle assignment, Alonso-Mora et al., 2017

Introduction: our approach

Our point of view

- Effect of the introduction of a ride sharing/taxi platform
- Game theory → predict the outcome

Original paper¹

- Impact of the introduction of a ride sharing platform
- Game theory: new model of the population

Extension

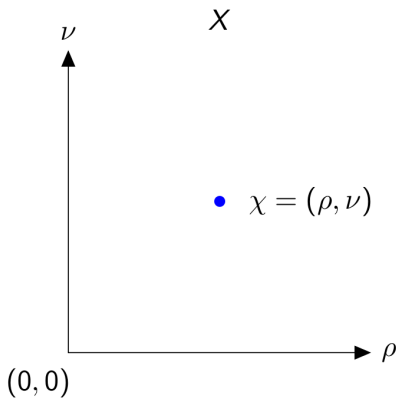
- What happens if a taxi platform competes with the ride sharing platform ?
- Impact on the revenue (can it increase ?)
- Original model + choice for the users

¹ Drivers, riders and service providers: the impact of the sharing economy on mobility, Courcoubetis et al., 2017

- 1 The Model
- 2 Theoretical analysis
- 3 Numerical analysis
- 4 Conclusion

The individuals

Society = {*Individual*}. Individuals have type $\chi \in X = \mathbb{R}_+^2$:



Nonatomic game:

- negligible impact of any individual
- continuum of players

$\rho > 0$ = utility for using private transportation

$\nu > 0$ = wage rate when working at a regular job

The platforms

The ride sharing platform

- Rental price r_1 :
- user(s), + riders
- Supply: from the population
- Demand: from the population

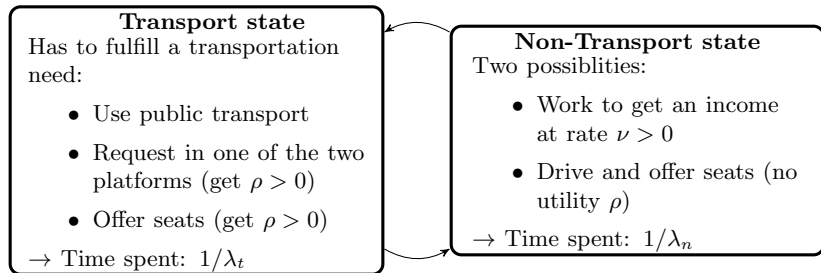
The taxi platform

- Rental price r_2 : - user(s)
- Supply: fixed number of taxis
 n_t
- Demand: from the population

Some other constants of the game:

- Number of seats per car: k
- Cost of ownership: ω
- Cost of usage: c

Individuals' possibilities



Standardized time: $1/\lambda_n + 1/\lambda_t = 1$, $\lambda_t, \lambda_n > 1$.

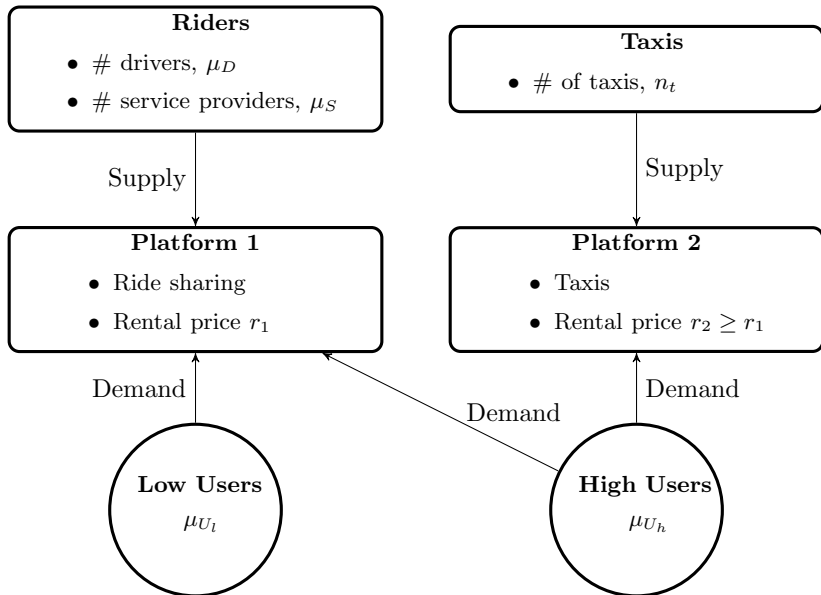
Strategies

Five strategies in $\Sigma = \{A, D, S, U_l, U_h\}$:

- Abstinent (A)
- Driver (D)
- Service Provider (S)
- Low User (U_l)
- High User (U_h)

For $\sigma \in \Sigma$, μ_σ : fraction of the population opting for strategy σ

Strategies (in the case $r_1 \leq r_2$)



Payoffs: $r = \min(r_1, r_2)$, $\bar{r} = \max(r_1, r_2)$

- $\pi_A(\rho, \nu) = \nu/\lambda_n$
- $\pi_D(\rho, \nu) = \nu/\lambda_n + \rho - \omega + k\bar{p}r_1 - c$
- $\pi_S(\rho, \nu) = \rho - \omega + \lambda_t(k\bar{p}r_1 - c)$
- $\pi_{U_l}(\rho, \nu) = \nu/\lambda_n + p_l(\rho - r)$
- $\pi_{U_h}(\rho, \nu) = \nu/\lambda_n + p_l(\rho - r) + (1 - p_l)p_h(\rho - \bar{r})$

p_l, p_h, \bar{p} : probabilities, depends on the distribution

$\mu = (\mu_A, \mu_D, \mu_S, \mu_{U_l}, \mu_{U_h})$.

Nash Equilibrium

Informal definition

A situation where it is not in the interest of any player to unilaterally change his strategy

At equilibrium:

- Strategy of players given by $\sigma^* : X \rightarrow \Sigma$
- $\forall \chi = (\rho, \nu) \in X, \forall \sigma \in \Sigma, \pi_{\sigma^*(\chi)}(\rho, \nu) \geq \pi_{\sigma}(\rho, \nu)$
- X partitionned into sets
 $P_{\sigma} = \{\text{Player choosing strategy } \sigma\}, \sigma \in \Sigma$

Example of equilibrium

$c=0.4, o=0.1, nt=0.5, r1=0.8, r2=0.9$

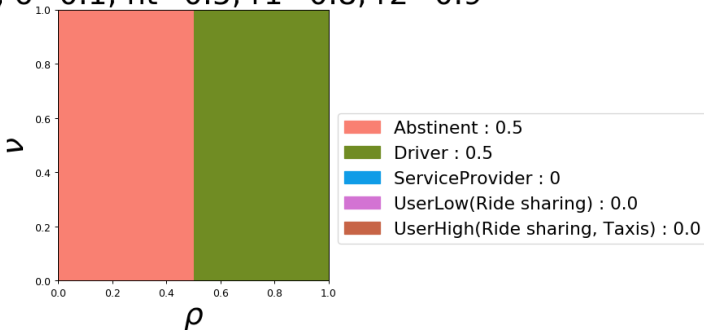


Figure: Equilibrium with parameters $\lambda_t = 6, k = 2$ (o stands for ω)

Outline

- 1 The Model
- 2 Theoretical analysis**
- 3 Numerical analysis
- 4 Conclusion

The revenue of the ride sharing platform: \mathcal{R}

Proportionate to:

- The rental price r_1
- The number of seats sold (depends on which is the cheapest platform)

If $r_1 \leq r_2$:

$$\mathcal{R} = r_1 \times p_l \times (\mu_{U_l} + \mu_{U_h})$$

If $r_1 > r_2$:

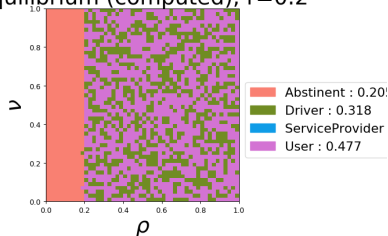
$$\mathcal{R} = r_1 \times p_h \times (1 - p_l) \mu_{U_h}$$

Some results: when $r_1 \leq r_2$ ($r = r_1$)

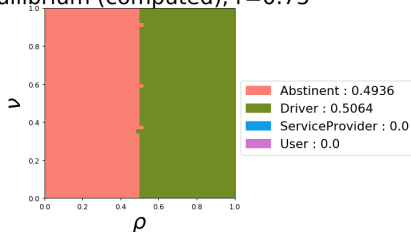
Theorem

If $r_1 \geq \frac{\omega+c}{k+1}$, then the equilibrium is the same as in the original game (without taxis).

Equilibrium (computed), $r=0.2$



Equilibrium (computed), $r=0.75$



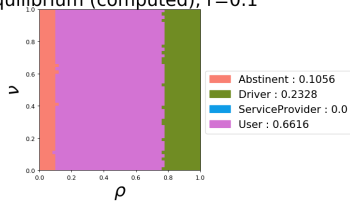
$$\omega = 0.1, c = 0.4, k = 2, n_t = 0.1, \lambda_t = 6.$$

Some results: when $r_1 \leq r_2$ ($r = r_1, r_t = r_2$)

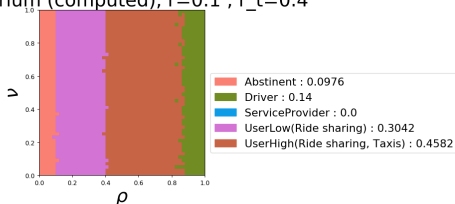
Theorem

If $\omega \leq c/k$ then adding the taxi platform can not increase the revenue of the ride sharing platform

Equilibrium (computed), $r=0.1$



Equilibrium (computed), $r=0.1, r_t=0.4$



$$\omega = 0.1, c = 0.4, k = 2, n_t = 0.1, \lambda_t = 6.$$

Some results: The revenue can increase

Theorem

There exists some values of our parameters for which the revenue of the ride sharing platform strictly increases

Equilibrium (computed), $r=0.55$, $r_t=0.4$

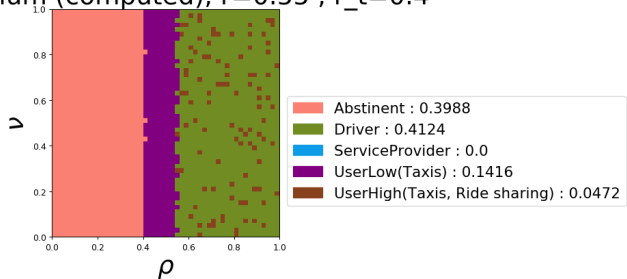


Figure: Equilibrium with parameters $\omega = 0.1$, $c = 0.4$, $k = 2$, $n_t = 0.1$, $\lambda_t = 6$.

Outline

- 1 The Model
- 2 Theoretical analysis
- 3 Numerical analysis**
- 4 Conclusion

The Best Response Dynamics Algorithm

Algorithm 1 Best Response Dynamics

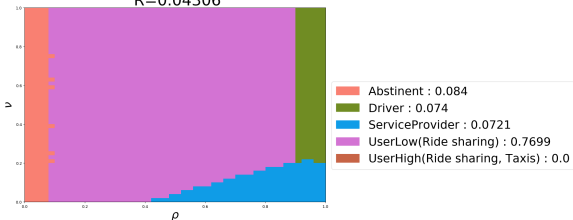
- 1: Each player is assigned a strategy randomly
 - 2: **while** a player changed strategy **do**
 - 3: **for** each player **do**
 - 4: Choose the payoff-maximizing strategy;
 - 5: Update distribution
-

This algorithm:

- Works on a large number of players (5000)
- Does not necessarily converge
- When it does, we have a Nash Equilibrium

Better revenue

$c=0.05, o=0.4, lt=6.0, r1=0.08500, r2=0.86000$
 $R=0.04306$



$nt=1.0, R=0.04406$

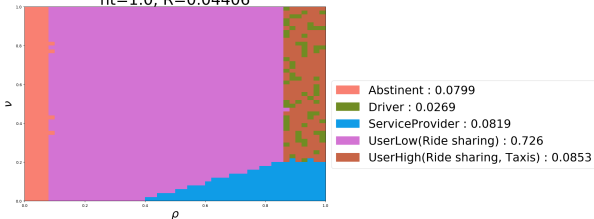


Figure: Equilibria without (top) and with (bottom) taxis, for $k = 1$

Price dynamics

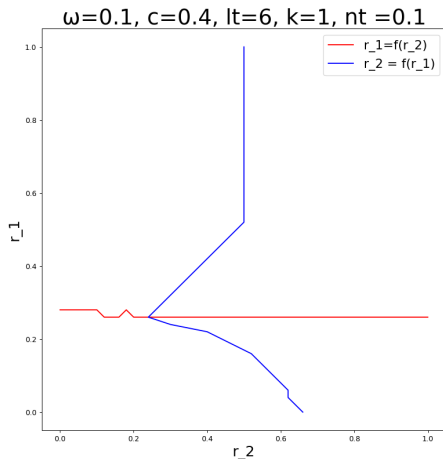


Figure: Optimizing price of one platform as a function of the price of the other platform

Outline

- 1 The Model
- 2 Theoretical analysis
- 3 Numerical analysis
- 4 Conclusion**

Conclusion and future work

- Model difficult to study: the model changes if $r_1 \leq r_2$ or if $r_2 > r_1$
- However, we do have some results:
 - Conditions that ensure that the revenue does not increase
 - Numerical/Analytical example of an increasing revenue
 - Situations that do not change by adding taxis

Future possibilities

- Condition of existence of service providers (independent of the distribution)
- Study the price dynamics: numerical simulations may suggest what happens

Other functions of interest: definition

- Distribution: $\mu_\sigma(\mathbf{s}) = \int_X \delta_{\mathbf{s},\sigma} \mathbf{d}\chi$
- Ownership: $\Omega(\mu) = \mu_S + \mu_D$;
- Traffic intensity: $\Gamma(\mu) = \mu_S + \mu_D/\lambda_t$;
- Social Welfare: $W(\mathbf{s}) = \sum_{\sigma \in \Sigma} \int_X \pi_\sigma(\chi) \cdot \delta_{\mathbf{s},\sigma} \mathbf{d}\chi$.

Other functions of interest: curves

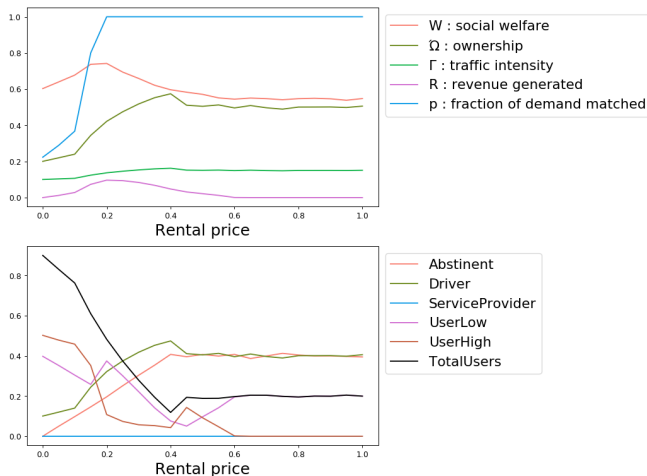


Figure: Curves with parameters $\omega = 0.1$, $c = 0.4$, $k = 2$, $n_t = 0.1$, $\lambda_t = 6$.

The matching functions

If $r_1 \leq r_2$:

- $p_l = \frac{k(\mu_D + \lambda_t \mu_S)}{\mu_{U_l} + \mu_{U_h}} \wedge 1$

- $p_h = \frac{n_t}{(1-p_l)\mu_{U_h}} \wedge 1$

- $\bar{p} = \frac{\mu_{U_l} + \mu_{U_h}}{k(\mu_D + \lambda_t \mu_S)} \wedge 1$

If $r_1 > r_2$:

- $p_l = \frac{n_t}{\mu_{U_l} + \mu_{U_h}} \wedge 1$

- $p_h = \frac{k(\mu_D + \lambda_t \mu_S)}{(1-p_l)\mu_{U_h}} \wedge 1$

- $\bar{p} = \frac{(1-p_l)\mu_{U_h}}{k(\mu_D + \lambda_t \mu_S)} \wedge 1$