Musing about Henri Poincaré's "Mathematical Creation": Fractal Jigsaw Puzzles and Computational Aesthetics

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"Mathematical Creation"

H. Poincaré, 1910. Mathematical Creation. The Monist, Vol. XX.

- A classical essay on creativity in the work of a mathematician.
- Short (10 pages).
- I will comment on it from my own mathematical perspective.

I thank Joanna R-L for inspiring me to do this creative work!

Henri Poincaré

Jules Henri Poincaré (1854–1912) was a French mathematician, theoretical physicist, engineer, and philosopher of science.

In his research on the three-body problem, Poincaré became the first person to discover a chaotic deterministic system. He is also considered to be one of the founders of the field of topology.

The first sentence of the essay

The genesis of mathematical creation is a problem which should intensely interest the psychologist. It is the activity in which the human mind seems to take least from the outside world, in which it acts or seems to act only of itself and on itself, so that in studying the procedure of geometric thought we may hope to reach what is most essential in man's mind.

Poincaré envisions cognitive science! (And artificial intelligence?)

The main theses of "Mathematical creation"

The nature of math creativity:

— Mathematics is hard since we get distracted.

— But good memory and attention don't explain math talent.

— Useful reasonings are extremely rare.

— Math talent is seeing a reasoning as a whole. *(Gestalt)*

— Math creation is a recombination foreseeing the latent order.

The nature of illuminations:

— Illuminations arise in a mathematician's work.

— They require prior conscious effort, however.

— Illuminations are not a psychic superpower!

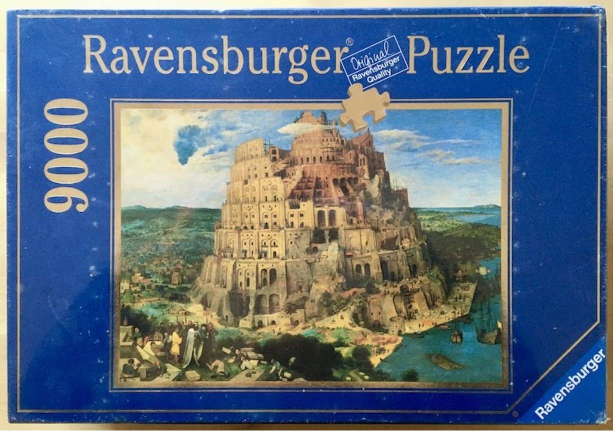
— They can be false and are usually incomplete.

— Illuminations always appeal to the aesthetic sense.

My interpretation

(somewhat outrageous)

The Tower of Babel — the first inspiration



In my childhood, I did a lot of jigsaw puzzles with my mother.
Bruegel's Tower of Babel of 9000 pieces was the largest one.

The jigsaw puzzle metaphor

Math as a jigsaw puzzle: (Or a tangram puzzle?)

Doing mathematics is like a jigsaw puzzle with a hidden image:
definitions, theorems, and proofs — freedom of form and topic.

Correct puzzle combinations are extremely rare!!!

— They cannot be found by a random or exhaustive search.

There is less randomness in puzzles than seen superficially:

— When we only see shapes of puzzle pieces, we arrange very slow.
(time complexity: exponential in the number of pieces)

— When we foresee the image on the pieces, we arrange faster!
(complexity: quadratic $\times$ the number of foreseen images)

How many distinct images can we foresee?

Fractals — the second inspiration

Fractals are (beautiful) objects with an (approximate) self-similarity.

— The total number of random objects is exponential in the resolution.
— The total number of random fractal objects is stretched exponential in the resolution. (Much smaller than of random objects!)

The fractal jigsaw puzzle hypothesis (beauty is rare!)

Useful mathematics is a fractal:

— It's as if there was a neat fractal image on the puzzle pieces.

— The total number of fractal images is stretched exponential.

— Thus the search complexity drops from exponential to quadratic $\times$ stretched exponential.

Illuminations are foreseen hypothetical images:

— They are incomplete since it requires conscious checking whether pieces can be aligned to them actually.

— They are aesthetically appealing since fractals are beautiful.

— They are not superpowers if sensing math beauty is effective.

If sensing math beauty is effective, can we learn it?

— To a certain extent, yes. (Appropriate teacher and time.)

— But mathematical imagination is laboriously constructed!

Recap: Towards computational aesthetics of math

Is beauty a proxy measure of soundness $\times$ usefulness?

— Paul Erdős often referred to "The Book" in which God keeps the most elegant proof of each mathematical theorem.
(M. Aigner and G. Ziegler wrote a book "Proofs from the Book".)

Education:
— Can we teach the mathematical beauty to schoolchildren?

Artificial intelligence:
— Can automated theorem provers foresee the math beauty?
(Large language models can actually guide theorem provers.)

Philosophy:
— Can we only do beautiful math, ugly one remaining unknown?
(The case of the four-color map theorem.)