On the Process of Becoming a Mathematician: A Personal Account

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Who am I?

- I am a 49yo associate professor working in information theory and statistical models of language.
- I am a member of both the Polish Mathematical Society and the International Quantitative Linguistic Association.
- Formally, I did my PhD in computer science understood as a branch of mathematical sciences.
- Earlier, I obtained my MSc in theoretical physics (statistical mechanics) from Warsaw University.
- Yet earlier, I attended I LO im. M. Kopernika in Łodź — a high school that produced a score of olympiad winners.

My path to become a mathematician wasn't straight and I am sure that it will never be fully complete in the sense that I abandon worldly matters in favor of pure abstraction.

A curious child

- I was a curious child. One of my first memories is touching the hot iron with my flat palm to check if it was really hot.
- I learned reading early and was an avid book moth in glasses. My dad forbade me to read Feynman's Physics Lectures when I was 7yo or so.
- I studied available sources in various disciplines sequentially: dinosaurs first, then maps, encyclopedias, astronomy, chemistry, and biology pop-sci, random phrasebooks.
- My mother called me at some point a cabinet naturalist.
- She wanted me to learn English, French, Russian, and German. I disliked learning lexicons but devoured grammar sections. I invented toy grammars and loved Esperanto.
- With Lego blocks, I once made a mechanical rectifier.
- My mom noticed that I was in search for structures.

Why did I do this?

- As I remember, I was driven by extreme emotions:
  positive excitation of learning and negative fear of losing.
  — This fear blocked me at several important times in my life!
  In my 30s, I learned that I had a bipolar disorder.
- I refused playing chess because my father would always win.
- My dad and two uncles were full professors in spe.
  I both felt inferior and didn't see my future but in science.
- The open question remained: Which science?
- I probed many of them systematically.

High school

- My math professor Olga Stande authored our textbook.
  She was phenomenal, celebrated her 100th birthday recently.
- She also gave my an F for the first classwork.
- I had great teachers and plenty of learning activities besides
  (e.g. a private course in group theory by a classmate's dad).
- Teachers wanted me to participate in competitions.
- I participated in the mathematics and physics competitions,
  got to the national level in physics but a vicious cycle of
  negative emotions devoured me and I got no trophy.
- I was generally eager to learn anything but literature and
  history. (Martyrology and dates were beyond my nerves.
  Of language and history, I preferred historical grammar books.)
- My final high school exam in Polish was a mission impossible:
  I learnt nothing but principles of poem analysis, got A in
  written exam for such analysis, and was exempt from oral one.

Why master's in physics?

- It was hard to choose the domain of my master studies.
- I was definitely interested in hard sciences: fundamental laws of nature, mathematical structures, grammars of languages.
- I told my dad, an engineer: "I don't want to study pure math because I will not know where to draw new ideas from."
  (Mention a question to Flemming Topsøe.)
- Physics seemed the main sort of applied math available at the university. I loved combining math with the real world.
- It was somewhat before the computer science boom. Email was there but WWW appeared during my studies.

Nerves, nerves, nerves!

- I had an unpleasant examiner in mathematical analysis.
  He wanted to fail me because I hadn't learn systematically
  but he mentioned I had "ideas" to the supervising professor.
  — The latter said "Good!", smiled, and granted me an A.

- This happened twice. Awaiting a similar situation for the third
  time, I tried to quit physics for psychology but I was stopped
  by a friendly supervisor. Weak nerves again!

From micro to macro

- I began my physics studies with a strong interest in fundamental forces and elementary particles.
- Once I learned about statistical mechanics, my interest shifted towards explaining the macroscale from the microscale. (Estate Khmaladze: "What is the difference between water and watermelon from a statistical point of view?")
- I entertained ideas such as fluid mechanics, deterministic chaos, and emergence of probabilities from Newtonian laws.
- I wrote my master thesis on theoretical derivation of phase transitions of a gas absorbed on the surface of a crystal.
- My supervisor Marek Napiórkowski wanted me to pursue a PhD program but I preferred to take on my parallel interests in languages and their grammars.

Computational linguistic affair

- By the end of my master's, I have learned that there exist computational linguists and I started following them.
- After a year of busy unemployment, I got hired at the Institute of Computer Science PAS.
- My task was to program a part-of-speech tagger of the first corpus of Polish, by combining linguistics and statistics.
- I was to learn programming, a skill quite different to manipulating integrals and matrices. Debugging taught patience and required tempered emotions, which wasn't easy.
- In parallel, I learned of Zipf's law and Hilberg's hypothesis, which struck me and drew my interest as a former physicist.

Towards mathematics of information

- Hilberg's hypothesis implies that the mutual information between the past and the future of a text grows unboundedly.

- I attended Fred Jelinek's lecture on statistical NLP in Prague. I tried to communicate him my ideas about Hilberg's hypothesis but I couldn't word my vague thoughts properly.

- Later I contacted a few other researchers abroad about Hilberg's hypothesis and I got invited by Jim Crutchfield to the Santa Fe Institute for a month.

- In a sudden illumination there, late in the night, I invented the Santa Fe process, a simple example that was supposed to visualize Hilberg's hypothesis. I felt this intuitively but I desperately needed advanced math to prove it.

- I came back to Poland with a strong resolution to learn advanced probability and theorem proving.

Two mentors

- Once the corpus of Polish had been tagged, I got to my proper PhD research in mathematics of Hilberg's hypothesis.
- At ICS PAS, I had two mentors: Jan Mielniczuk, the PhD supervisor, and Jacek Koronacki, the director.
- Their personalities were like the good and the bad policemen: the former criticizing and the latter praising. This turned out very stimulating. Both had important content words to convey and were very supportive despite their tones were predictable!

Learning mathematics

- My resolution to learn math wasn't easy to carry out.
- I had a tendency to "read" books by carrying them in my bag.
- I liked reinventing things on my own.
- Independent thinking is necessary to become creative
  but it can also slow down research if it boils down to the
  reiterated discovery of things already known.

Theorem proving and writing math

- Learning to prove theorems and structurize my thoughts into definitions, examples, and propositions was difficult since it required to express oneself in natural language as if it were a mix of a programming language and legal writing.

- **Why bother?**
  Because we want to propagate verifiable complex mental images in other people's minds.

- Imagine that you can steer your reader in a precise way
  — unless you make her lose track of your ideas
  — by writing in an either wrong, boring, or obfuscated way!
     (Writing law requires heart, according to my mom's friend.)

- I had to relearn writing in English or in Polish and to debug my writing patiently to sound both formal and readable.
              (Are we living in an illusion of mutual understanding? :)

Constructing imagination/intuition

- Advanced probability, notably measure theory, requires also imagining actual infinities and irregular shapes.
- Imagination or intuition has to be constructed and gradually extended by (counter)-intuitive (counter)-examples.
- There are two systems in action:

  - **Intuition — thinking fast — generating ideas.**
  - **Deduction — thinking slow — checking ideas.**

- Deduction has but a few rules, it is intuition that is hard! (Although it appears the other way round.)
- Intuition can be perceived as subjective probabilities, namely, seeing some choices as obvious and other as cumbersome.
- It is intuition combined with the sanity check of deduction that gives you the fluent feeling of understanding.

Switching among different views

- Math requires combining different images and metaphors:

  - **numbers,**
  - **symbols,**
  - **geometry,**
  - **graphs,**
  - **language,**
  - **programming.**

- The power comes from switching among these perspectives, not necessarily from sticking to one particular mental image.

Example I: Probability

Particular instances of the same concept:

- Empirical frequency of a repeatable phenomenon. (frequency)
- Limit of such frequencies. (frequency & extrapolation)
- Subjective and evolving belief of a learning agent. (Bayesian)
- Propensy of an unpredictable phenomenon. (randomness)
- Weight in a weighted mean. (abstract)
- Fraction of favorable elementary events. (abstract)
- Relative volume of a figure. (abstract & geometry)
- Result of a smoothing procedure. (computation)
- Output of a complicated black box. (computation)

Example II: Complex numbers
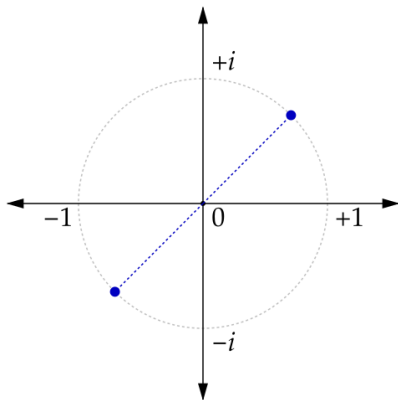
Complex numbers ($\boldsymbol{a + bi}$) come from combining algebra with planar geometry. They can be used to derive trigonometric formulas and yield concepts as powerful as contour integrals and probability amplitudes. They are useful in modeling classical fluids and quantum mechanics.

Post-doc years

- I defended my PhD on excess entropy of stationary processes.
- For post-docs, I went to UNSW Sydney and CWI Amsterdam.
- The next few years I spent in Poland due to the mental crisis (an acute psychosis during wedding & later 3-year depression). — I learnt many unexpected things about the limits of myself.
- But I took pills and worked all the time. Instead of going to conferences, I preferred to write journal papers and books. I obtained the habilitation and got a permanent position.
- I went to Japan as a visiting professor — also to test if my mental health permits.
- My first PhD student defended four years ago.
- I still learn how to write and speak about math and life.

Some texts on doing mathematics

- Henri Poincaré, Mathematical Creation. The Monist, Vol. XX, 1910.
- G. H. Hardy, A Mathematician's Apology. Cambridge University Press, 1940.
- Imre Lakatos, Proofs and Refutations: The Logic of Mathematical Discovery. Cambridge University Press, 1976.
- Clive Newstead, An Infinite Descent into Pure Mathematics. 2022. (`https://infinitedescent.xyz`)
- David Bessis, Mathematica: A Secret World of Intuition and Curiosity. Yale University Press, 2024.
- Jeremy Avigad, Patrick Massot, Mathematics in Lean, 2024. (`https://leanprover-community.github.io/`)
- Albert Jiang, Language models for verifiable mathematical automation. (PhD thesis, coming soon)