

An Approach to Making Decisions with Metaset

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Abstract. The metaset is a new approach to sets with partial membership relation. Metasets are designed to represent and process vague, imprecise data, similarly to fuzzy sets or rough sets. They make it possible to express fractional certainty of membership, equality, and other relations. In this paper we demonstrate an example of the application of first-order metasets to solving the problem of finding the most appropriate holiday destination for a tourist, taking his preferences into account. The imprecise idea of ‘a perfect holiday destination’ is represented as a metaset of places whose membership degrees in the metaset are interpreted as their qualities. Client preferences are functions which enable real-number evaluation of the subjective rating of a given destination

Keywords: metaset, partial membership, set theory

1 Introduction

The metaset is a new attempt at defining the notion of sets with partial membership relation [10]. Metasets enable the representation and processing of vague, imprecise data, similarly to fuzzy sets [16] or rough sets [8]. In addition to fractional certainty of membership, equality or other set-theoretic relations and their negations, metasets admit a hesitancy degree of membership [11,14,13], similarly to intuitionistic fuzzy sets [1]. The general idea of the metaset is inspired by the method of forcing in classical set theory [2]. Despite these abstract origins, the definitions of metaset and related notions (i.e. set-theoretic relations or algebraic operations [12]) are directed towards efficient computer implementations and applications [9].

The following paper introduces an example of the application of this new approach. There are many real-life problems which require tools for modelling fractional satisfaction of some properties. They are usually solved by modelling these properties with the ‘fuzzy’ membership relation of a fuzzy or rough set. Here we try another theory, that of metasets. Since the set of membership values for metasets constitutes a partial order (in fact it is a Boolean algebra), there is great potential here for modelling of imprecise phenomena.

The specific contribution of this paper is to show how metasets can be applied to (software) tools which support decision-making. The problem we have

selected is evaluation of the attractiveness of tourist destinations. These may be destinations that the user is considering in a tourist office as the best location for a holiday, but they may also be tourist attractions located in one city, such as monuments, galleries, museums, etc. In both cases, the problem lies in the selection of one or more sites from among numerous proposals. The input in this problem is a list of sites with the location and a brief description of each. The output has to be a numeric score assigned to each location that allows us to compare them and ultimately select the best one. Difficulties that appear here include the following: (a) to select the most important attributes from the description of the site, (b) to personalize tourist preferences and (c) to assign a score that differentiates the locations in terms of tourist needs. In this paper we show how to use metaset to describe tourist preferences and how this representation helps to compute the degree of membership of a particular object in a set of perfect holiday destinations. It is emphasized that this degree will be different for different types of tourists and will be closely related to their preferences.

The proposed approach can be used in automated personalized tour-planning devices. In particular, it can be used in solving Tourist Trip Design Problems, TTDP (see e.g. [15]). The starting point in the TTDP is the orienteering problem (see e.g. [5,6]). In this problem a set of nodes is given, each with a score. The goal is to determine a path, limited in length, that visits some nodes and maximizes the sum of the collected scores. However, before the solution to this problem is presented, values must be assigned to the nodes. This is where the algorithm we propose may be helpful. If the nodes represent locations, then by using metaset we can calculate scores which represent the interest of a given tourist.

The remainder of the paper is structured as follows. Section 2 gives the theoretical background, i.e., we briefly recall the main definitions and lemmas concerning metaset. Section 3 presents the problem of assigning to tourist locations an evaluation of their attractiveness and its solution in terms of metaset. Section 4 provides a generalization of the concept introduced. Conclusions are given in Section 5.

2 Metaset

A metaset is a classical crisp set with a specific internal structure which encodes the membership degrees of its members. Therefore, all Zermelo-Fraenkel [3,7] axioms apply to metaset as well. The membership degrees are expressed as nodes of the binary tree \mathbb{T} . All the possible membership values make up a Boolean algebra. They can be evaluated as real numbers as well.

There are several fundamental notions associated with metaset which we recall in this section. These allow the membership relation to be defined and evaluated. We then use it to model the quality of tourist destinations and a client's preferences.

2.1 Basic Definitions

For simplicity, in this paper we deal only with first-order metasets.³ A metaset of this type is a relation between some set and the set of nodes of the binary tree \mathbb{T} . Thus, the structure we use to encode the degrees of membership is based on ordered pairs. The first element of each pair is the member and the second element is a node of the binary tree which contributes to the membership degree of the first element.

Definition 1. A set which is either the empty set \emptyset or which has the form:

$$\tau = \{ \langle \sigma, p \rangle : \sigma \text{ is a set, } p \in \mathbb{T} \}$$

is called a first-order metaset.

The class of first-order metasets is denoted by \mathfrak{M}^1 . The binary tree \mathbb{T} is the set of all finite binary sequences, i.e., functions whose domains are finite ordinals, valued in 2:⁴

$$\mathbb{T} = \bigcup_{n \in \mathbb{N}} 2^n . \tag{1}$$

The ordering \leq in the tree \mathbb{T} (see Fig. 1) is the reverse inclusion of functions: for $p, q \in \mathbb{T}$ such that $p: n \mapsto 2$ and $q: m \mapsto 2$, we have $p \leq q$ whenever $p \supseteq q$, i.e., $n \geq m$ and $p \upharpoonright_m = q$. The root $\mathbb{1}$ being the empty function is the largest element of \mathbb{T} in this ordering. It is included in each sequence and for all $p \in \mathbb{T}$ we have $p \leq \mathbb{1}$.

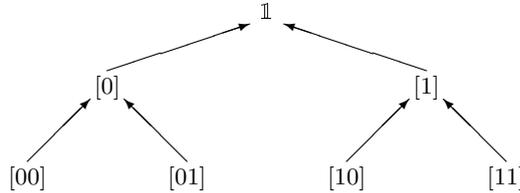


Fig. 1. The levels \mathbb{T}_0 - \mathbb{T}_2 of the binary tree \mathbb{T} and the ordering of nodes. Arrows point at the larger element.

We denote binary sequences which are elements of \mathbb{T} using square brackets, for example: $[00]$, $[101]$. If $p \in \mathbb{T}$, then we denote its children with $p \cdot 0$ and $p \cdot 1$. A *level* in \mathbb{T} is the set of all finite binary sequences with the same length. The set 2^n consisting of sequences of the length n is the level n , denoted by \mathbb{T}_n . The level 0 consists of the empty sequence $\mathbb{1}$ only. A *branch* in \mathbb{T} is an infinite binary

³ See section 4 for the introduction to metasets in general.
⁴ For $n \in \mathbb{N}$, let $2^n = \{ f: n \mapsto 2 \}$ denote the set of all functions with the domain n and the range $2 = \{ 0, 1 \}$ – they are binary sequences of the length n .

sequence, i.e., a function $\mathbb{N} \mapsto 2$. Abusing the notation we will write $p \in \mathcal{C}$ to mark, that the binary sequence $p \in \mathbb{T}$ is a prefix of the branch \mathcal{C} . A branch intersects all levels in \mathbb{T} , and each of them only once.

Since a metaset is a relation, we may use the following standard notation. For the given $\tau \in \mathfrak{M}^1$, the set $\text{dom}(\tau) = \{ \sigma : \exists p \in \mathbb{T} \langle \sigma, p \rangle \in \tau \}$ is called the *domain* of the metaset τ , and the set $\text{ran}(\tau) = \{ p : \exists \sigma \in \text{dom}(\tau) \langle \sigma, p \rangle \in \tau \}$ is called the *range* of the metaset τ .

A metaset is *finite* when it is finite as a set of ordered pairs. Consequently, its domain and range are finite. The class of finite first-order metasets is denoted by $\mathfrak{M}\mathfrak{F}^1$. Thus,

$$\tau \in \mathfrak{M}\mathfrak{F}^1 \quad \text{iff} \quad |\text{dom}(\tau)| < \aleph_0 \wedge |\text{ran}(\tau)| < \aleph_0 . \quad (2)$$

This class is particularly important for computer applications where we deal with finite objects exclusively.

2.2 Interpretations

An interpretation of a first-order metaset is a crisp set. It is produced out of a given metaset using a branch of the binary tree. Different branches determine different interpretations of the metaset. All of them taken together make up a collection of sets with specific internal dependencies, which represents the source metaset by means of its crisp views. Properties of crisp sets which are interpretations of the given first-order metaset determine the properties of the metaset itself. In particular we use interpretations to define set-theoretic relations for metasets.

Definition 2. *Let τ be a first-order metaset and let \mathcal{C} be a branch. The set*

$$\tau_{\mathcal{C}} = \{ \sigma \in \text{dom}(\tau) : \langle \sigma, p \rangle \in \tau \wedge p \in \mathcal{C} \}$$

is called the interpretation of the first-order metaset τ given by the branch \mathcal{C} .

An interpretation of the empty metaset is the empty set, independently of the branch.

The process of producing an interpretation of a first-order metaset consists in two stages. In the first stage we remove all the ordered pairs whose second elements are nodes which do not belong to the branch \mathcal{C} . The second stage replaces the remaining pairs – whose second elements lie on the branch \mathcal{C} – with their first elements. As the result we obtain a crisp set contained in the domain of the metaset.

Example 1. Let $p \in \mathbb{T}$ and let $\tau = \{ \langle \emptyset, p \rangle \}$. If \mathcal{C} is a branch, then

$$\begin{aligned} p \in \mathcal{C} &\rightarrow \tau_{\mathcal{C}} = \{ \emptyset \} , \\ p \notin \mathcal{C} &\rightarrow \tau_{\mathcal{C}} = \emptyset . \end{aligned}$$

Depending on the branch the metaset τ acquires one of two different interpretations: $\{ \emptyset \}$ or \emptyset . Note, that $\text{dom}(\tau) = \{ \emptyset \}$.

As we see, a first-order metaset may have multiple different interpretations – each branch in the tree determines one. Usually, most of them are pairwise equal, so the number of different interpretations is much less than the number of branches. Finite first-order metasets always have a finite number of different interpretations.

2.3 Partial Membership

We use interpretations for transferring set-theoretic relations from crisp sets onto metasets.⁵ In this paper we discuss only the partial membership.

Definition 3. *We say that the metaset σ belongs to the metaset τ under the condition $p \in \mathbb{T}$, whenever for each branch \mathcal{C} containing p holds $\sigma_{\mathcal{C}} \in \tau_{\mathcal{C}}$. We use the notation $\sigma \epsilon_p \tau$.*

Formally, we define an infinite number of membership relations: each $p \in \mathbb{T}$ specifies another relation ϵ_p . Any two metasets may be simultaneously in multiple membership relations qualified by different nodes: $\sigma \epsilon_p \tau \wedge \sigma \epsilon_q \tau$. Membership under the root condition $\mathbb{1}$ resembles the full, unconditional membership of crisp sets, since it is independent of branches.

The conditional membership reflects the idea that an element σ belongs to a metaset τ whenever some conditions are fulfilled. The conditions are represented by nodes of \mathbb{T} . There are two substantial properties of this technique exposed by the following two lemmas.

Lemma 1. *Let $\tau, \sigma \in \mathfrak{M}^1$ and let $p, q \in \mathbb{T}$. If $\sigma \epsilon_p \tau$ and $q \leq p$, then $\sigma \epsilon_q \tau$.*

Proof. If \mathcal{C} is a branch containing q then also $p \in \mathcal{C}$. Therefore $\sigma_{\mathcal{C}} \in \tau_{\mathcal{C}}$.

Lemma 2. *Let $\tau, \sigma \in \mathfrak{M}^1$ and let $p \in \mathbb{T}$. If $\forall_{q < p} \sigma \epsilon_q \tau$, then $\sigma \epsilon_p \tau$.*

Proof. If $\mathcal{C} \ni p$, then it also contains some $q < p$. Therefore, $\sigma_{\mathcal{C}} \in \tau_{\mathcal{C}}$.

In other words: $\sigma \epsilon_p \tau$ is equivalent to $\sigma \epsilon_{p \cdot 0} \tau \wedge \sigma \epsilon_{p \cdot 1} \tau$, i.e., being a member under the condition p is equivalent to being a member under both conditions $p \cdot 0$ and $p \cdot 1$, which are the direct descendants of p . Indeed, by lemma 1 we have $\sigma \epsilon_p \tau \rightarrow \sigma \epsilon_{p \cdot 0} \tau \wedge \sigma \epsilon_{p \cdot 1} \tau$. And if $\sigma \epsilon_{p \cdot 0} \tau$, then again, by lemma 1 we have $\forall_{q \leq p \cdot 0} \sigma \epsilon_q \tau$, and similarly for $p \cdot 1$. Consequently, we have $\forall_{q < p} \sigma \epsilon_q \tau$ and by lemma 2 we obtain $\sigma \epsilon_{p \cdot 0} \tau \wedge \sigma \epsilon_{p \cdot 1} \tau \rightarrow \sigma \epsilon_p \tau$.

Example 2. Recall, that the ordinal number 1 is the set $\{0\}$ and 0 is just the empty set \emptyset . Let $\tau = \{\langle 0, [0] \rangle, \langle 1, [1] \rangle\}$ and let $\sigma = \{\langle 0, [1] \rangle\}$. Let $\mathcal{C}^0 \ni [0]$ and $\mathcal{C}^1 \ni [1]$ be arbitrary branches containing $[0]$ and $[1]$, respectively. Interpretations are: $\tau_{\mathcal{C}^0} = \{0\}$, $\tau_{\mathcal{C}^1} = \{1\}$, $\sigma_{\mathcal{C}^0} = 0$ and $\sigma_{\mathcal{C}^1} = \{0\} = 1$. We see that $\sigma \epsilon_{[0]} \tau$ and $\sigma \epsilon_{[1]} \tau$. Also, $\sigma \epsilon_{\mathbb{1}} \tau$ holds.

Note, that even though interpretations of τ and σ vary depending on the branch, the metaset membership relation is preserved.

⁵ For the detailed discussion of the relations or their evaluation the reader is referred to [12] or [14].

2.4 Evaluating Membership

Membership degrees for metaset are expressed as nodes of \mathbb{T} . In fact, these nodes determine the basis of the Boolean Algebra of closed-open sets in the Cantor space 2^ω . Indeed, a $p \in \mathbb{T}$ is just a prefix for all infinite binary sequences which form a clopen subset of 2^ω . Thus, the membership relation for metaset is valued in the Boolean algebra. Nonetheless, for the sake of simplicity and in applications we usually refer to the binary tree when talking about membership.

In applications we frequently need a numerical evaluation of membership degrees. To define it first we consider the smallest subset of \mathbb{T} consisting of elements which determine the membership.

Definition 4. *Let $\sigma, \tau \in \mathfrak{M}^\perp$. The set*

$$\|\sigma \in \tau\| = \max \{ p \in \mathbb{T} : \sigma \epsilon_p \tau \}$$

is called the certainty grade for membership of σ in τ .

Note that by definition 3, $\|\sigma \in \tau\| = \max \{ p \in \mathbb{T} : \forall \mathcal{C} \ni p \ \sigma_{\mathcal{C}} \in \tau_{\mathcal{C}} \}$. Lemmas 1 and 2 justify definition 4. Indeed, if for $q \in \mathbb{T}$ the membership $\sigma \epsilon_q \tau$ is satisfied, which means that for any branch \mathcal{C} containing q it holds that $\sigma_{\mathcal{C}} \in \tau_{\mathcal{C}}$, then $q \in \{ p \in \mathbb{T} : \forall \mathcal{C} \ni p \ \sigma_{\mathcal{C}} \in \tau_{\mathcal{C}} \}$. Therefore, there exists a $p \in \|\sigma \in \tau\|$ such that $q \leq p$. And by lemma 1, each such p implies that $\sigma \epsilon_q \tau$, for $q \leq p$. This means that all the necessary membership information is contained in $\|\sigma \in \tau\|$. Moreover, no incorrect membership information can be inferred from this set. If for $r \leq s$ it is not true that $\sigma \epsilon_r \tau$, then $s \in \|\sigma \in \tau\|$ would contradict lemma 1. Note also that if $\sigma \epsilon_{q \cdot 0} \tau$ and $\sigma \epsilon_{q \cdot 1} \tau$, then consequently for any $r < q$ it holds that $\sigma \epsilon_r \tau$, and therefore by lemma 2 it holds that $\sigma \epsilon_q \tau$. Thus, the set of all $p \in \mathbb{T}$ such that $\sigma \epsilon_p \tau$ consists of subtrees whose roots are in $\|\sigma \in \tau\|$.

We define the numerical evaluation of membership by composing the membership function valued in 2^ω with the natural transformation $2^\omega \mapsto [0, 1]$ as follows.

Definition 5. *Let $\sigma, \tau \in \mathfrak{M}^\perp$. The following value is called the certainty value of membership of σ in τ :*

$$|\sigma \in \tau| = \sum_{p \in \|\sigma \in \tau\|} \frac{1}{2^{|p|}}.$$

Recall that $|p|$ is the length of the binary sequence p , which is equal to the number of the level containing p . One may easily see that $|\sigma \in \tau| \in [0, 1]$.

For the sake of the main topic of the discussion it is worth noticing that in the above definition we treat all the nodes within the same level uniformly, without distinguishing one from another. All nodes on the given level contribute the same factor of $\frac{1}{2^{|p|}}$ to the membership value. This will not be the case for the problem of evaluation of client preferences, where we modify this function.

3 The Problem

In this section we show how use metasets to solve the problem of assigning to tourist locations evaluations of their attractiveness. Here we will use a real data set from the city of Białystok. In the problem that we want to solve, a set of locations is given. We assume, that these are places that can be visited during a one-day trip. At this point we disregard the questions of how many places a tourist may wish to choose, the distance between them, and the means of arrival, focusing only on their attractiveness. The places we choose from can be divided into the following categories: (a) buildings (church, museum, gallery, etc.), (b) sports facilities (playgrounds, skate parks, etc.) and (c) outdoor places (park, forest, river, etc.).

Example 3. We selected attributes that may be important for the selection on the basis of interviews with potential tourists: (a) the possibility of eating something (*Fast food*, *Slow food*), (b) a place to sit and rest (*Physical rest*), (c) sports infrastructure (*Sports*), (d) the chance to explore the city (*Sightseeing*) and (e) a good place for hiking (*Hiking*). Classification of two locations, the Resort of Water Sports in Dojlidy (*RWS*) and Branicki Park & Planty (*PBP*), according to these attributes is shown in Table 1.

| | <i>Fast food</i> | <i>Slow food</i> | <i>Physical rest</i> | <i>Sports</i> | <i>Sightseeing</i> | <i>Hiking</i> |
|------------|------------------|------------------|----------------------|---------------|--------------------|---------------|
| <i>RWS</i> | YES | NO | YES | YES | NO | YES |
| <i>PBP</i> | YES | YES | YES | NO | YES | YES |

Table 1. Classification of locations

3.1 Modelling with Metasets

Our goal is to differentiate these tourist locations according to their attractiveness. To this end we build a binary tree designated by the expression ‘a perfect holiday destination’. For most tourists a good place for a holiday is somewhere they can relax and enjoy leisure activities (e.g., hiking, bike riding, sports). Therefore two branches extend from the root of this tree: *Relaxation* and *Activity* (see Fig. 2). Relaxation requires a convenient location (node: *Rest*) as well as a place for dining (node: *Food*). Places to rest include those that offer a respite for the body (beach, forest, etc.) (node: *Body*) or the soul (restaurant with live music, performances, etc.) (node: *Soul*). Among dining options we can distinguish between those which serve pizza, hamburgers or hot dogs (node: *Fast food*) and those that offer regional, organic, vegetarian cuisine, etc. (node: *Slow food*). Available activities can be classified as follows: sports (node: *Sports*) and tourism more generally (node: *Tourism*). Sports require infrastructure (node: *Infrastructure*). Sometimes it is also possible to take part in sporting

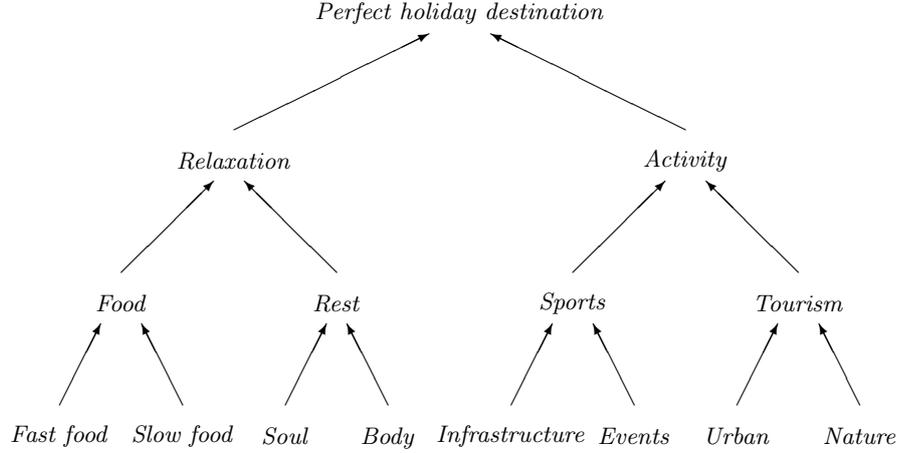


Fig. 2. The tree for *Perfect holiday destination*

events, such as a marathon (node: *Events*). Tourism can be in an urban area, and then includes visiting churches, galleries or architecturally interesting buildings (node: *Urban*). Some tourists prefer such activities as walking in the mountains or strolling through a park (node: *Nature*).

The remainder of the tree is built in a similar way. The greater the height of the tree, the more detailed the feedback. Taking into account a large number of attributes leads to more accurate assignment of locations to the tourist. The subtree for the *Infrastructure* node is depicted in Fig. 3. *Infrastructure* consists of locations such as swimming pools (node: *Locations*) and other facilities (node: *Facilities*). The most important of these are buildings (node: *Buildings*) and the possibility of buying (node: *Buy*) or renting (node: *Rent*) equipment. The tree we have built is only an example and can be changed depending on specific applications and needs.

Let us return to the tourist. First we define his expectations. We can do this using one of the following methods: (a) communication in natural language (a dialogue), (b) human-computer communication using an appropriate application (e.g. implementation of formal dialogue games [4]), or (c) a survey. To give an example, let us postulate two tourists with different expectations.

Example 4. Consider two sample tourists, *Ann* and *Ben*.

Ann is an active person. She likes sports, especially running. She strongly prefers nature to the city. She does not eat much, because she is constantly on a diet.

Ben prefers a relaxing holiday. Eating well is his highest priority. He does not like fast food. He enjoys bus tours and sightseeing. He does not like to play sports, but he is a fan of spectator sports. He enjoys watching concerts and shows.

We formalize the preferences of these tourists in example 5.

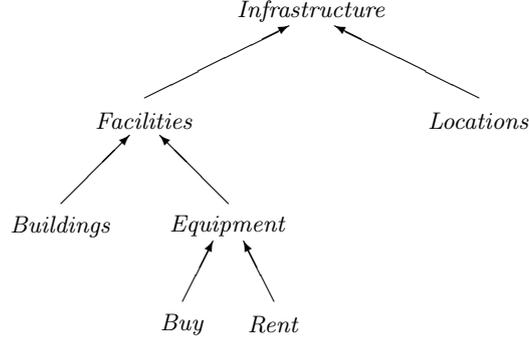


Fig. 3. Subtree for *Infrastructure* node

3.2 Evaluating Client Preferences

Definition 5 assumes uniform distribution of values throughout the nodes in \mathbb{T} : each $p \in \|\delta \in \Delta\|$ contributes the value of $\frac{1}{2^{|p|}}$ to $|\delta \in \Delta|$.

In the context discussed in the paper this might be interpreted as a client’s indifference as to what to choose: all possible choices represented as nodes within the same level are equally weighted.

For a $p \in \mathbb{T}$ both its children $p \cdot 0$ and $p \cdot 1$ contribute equally to the membership evaluation. Usually, however, clients have some preferences concerning places or activities and this preference may be expressed numerically.

To evaluate the quality of a destination taking client preferences into account we modify the definition 5 slightly to obtain an evaluation function which increases the impact of some nodes and decreases that of others. We build this function based on an interview with the client.

Definition 6. We define client preference to be a function $\mathbf{p}: \mathbb{T} \mapsto [0, 1]$ such that

$$\forall q \in \mathbb{T} \quad \mathbf{p}(q \cdot 0) + \mathbf{p}(q \cdot 1) = 1 . \tag{3}$$

and we take $\mathbf{p}(\mathbb{1}) = 1$ for the root.

Now we may evaluate the quality of the destination δ taking preferences \mathbf{p} into account to obtain the subjective value of the quality of the destination as follows:

Definition 7. Let δ be a destination and let Δ be a metaset of destinations. The \mathbf{p} -quality of the destination δ is the following value:

$$|\delta \in \Delta|_{\mathbf{p}} = \sum_{q \in \|\delta \in \Delta\|} \prod_{0 \leq i \leq |q|} \mathbf{p}(q_{\uparrow i}) .$$

The symbol $q_{\uparrow i}$, where $0 \leq i \leq |q|$ denotes all the consecutive prefixes of the binary sequence q , including the empty one (for $i = 0$) and q itself (for $i = |q|$, which is the length of the sequence q). Note that $q_{\uparrow |q|} = q$, since $\text{dom}(q) = |q|$ and $q_{\uparrow 0} = \emptyset = \mathbb{1}$.

The \mathbf{p} -quality of a destination reflects a client's preferences. For different clients with different \mathbf{p} preference functions it may result in different ratings for the given destination. We discuss this and present examples in the following section.

3.3 Solution to the Problem

To demonstrate the advantages of our approach we take into account the preferences of the clients mentioned in example 4 when comparing the two destinations defined in example 3. We have two sample locations with opposite characteristics: 'active' (*RWS*) and 'non-active' (*PBP*). There are also two clients: *Ann*, an active person, and *Ben*, who has a sedentary lifestyle. We show that the evaluated client preferences for particular locations are consistent with common sense: *Ann* prefers *RWS* while *Ben* prefers *PBP*. In particular, we claim that

$$|PBP \in \Delta|_{Ann} \leq |RWS \in \Delta|_{Ann} , \quad (4)$$

and

$$|RWS \in \Delta|_{Ben} \leq |PBP \in \Delta|_{Ben} . \quad (5)$$

The expression $|\delta \in \Delta|_X$ is the real number representing the quality of δ as a 'perfect holiday destination' (i.e. the membership value of δ in Δ), taking into account the preferences of the client X . The above formulas formally express the fact that *Ann* prefers active destinations and *Ben* non-active ones. In example 4 we expressed sample preferences using natural language. We now demonstrate the \mathbf{p} functions for these clients, constructed following a detailed investigation of their preferences (personal interview or computer-aided tool). The functions are depicted in Fig. 4 and Fig. 5.

Example 5. Ann prefers Activity to Relaxation. We found that this preference is expressed by the ratio 3/1, so we set $\mathbf{p}(\textit{Activity}) = 0.75$ and $\mathbf{p}(\textit{Relax}) = 0.25$ (see Fig. 4). She likes playing sports and having a good rest afterwards. She professed a ratio of 7/10 in favour of *Sports* over *Tourism*. Since she does not eat much, we assumed a ratio of 1/5 between eating and resting. She prefers *Nature* to *Urban* tourism. We assume here a ratio of 4/1 and therefore we set $\mathbf{p}(\textit{Nature}) = 0.8$ and $\mathbf{p}(\textit{Urban}) = 0.2$. We know that she rarely attends sporting events and therefore we set $\mathbf{p}(\textit{Events}) = 0.1$ and $\mathbf{p}(\textit{Infrastructure}) = 0.9$. For all other nodes $q \neq \mathbb{1}$ we set $\mathbf{p}(q) = 0.5$.

Let us now consider *Ben*. Since he dislikes any form of *Activity*, we assume $\mathbf{p}(\textit{Activity}) = 0.15$ and $\mathbf{p}(\textit{Relaxation}) = 0.85$. Eating well is *Ben*'s most significant preference, so we assume $\mathbf{p}(\textit{Food}) = 0.75$ and $\mathbf{p}(\textit{Rest}) = 0.25$. Because we know he values good food, we assume $\mathbf{p}(\textit{Fastfood}) = 0.05$ and $\mathbf{p}(\textit{Slowfood}) = 0.95$. *Ben*'s detailed preferences are depicted in Fig. 5.

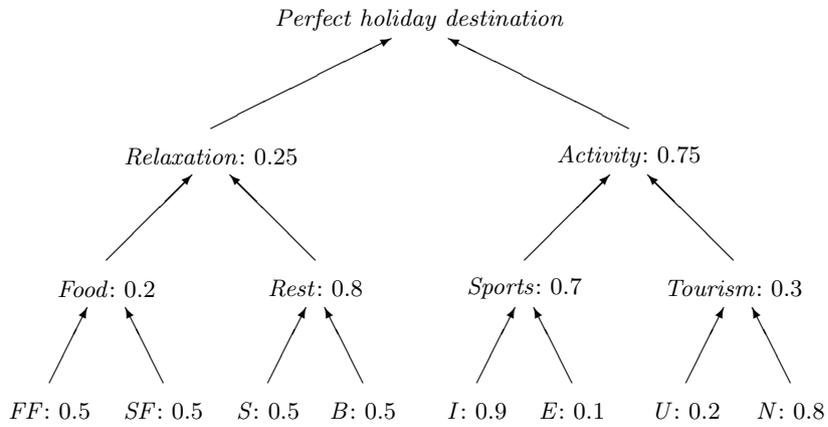


Fig. 4. Ann's preferences (we used abbreviations in the last level).

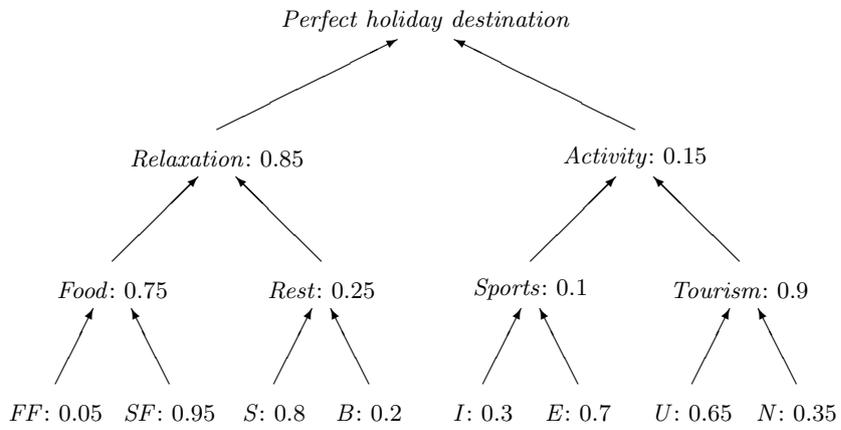


Fig. 5. Ben's preferences (we used abbreviations in the last level).

The value of 0.5 may be interpreted as indifference towards a particular choice in both cases.

We now show how the preferences of the clients in example 4 affect the subjective quality of a given destination. First, for reference, we calculate the numerical value of the membership, which also represents the preferences of a totally indifferent client.

We consider the two locations, *RWS* and *PBP*, with the following attributes (cf. Ex. 3).

$$RWS : Fast\ food, Body, Infrastructure, Nature , \quad (6)$$

$$PBP : Food, Soul, Urban . \quad (7)$$

First, let us calculate the degrees of membership of both places in the metaset Δ consisting of *perfect holiday destinations*. They tell us the measure of objective quality of these places, i.e. the degree to which the idea of a perfect holiday destination is satisfied by these particular destinations. These degrees are also equal to those resulting from evaluation of the preferences of a totally indifferent client.

$$\|RWS \in \Delta\| = \{ Fast\ food, Body, Infrastructure, Nature \} , \quad (8)$$

$$= \{ [000], [011], [100], [111] \} , \quad (9)$$

$$\|PBP \in \Delta\| = \{ Food, Soul, Urban \} , \quad (10)$$

$$= \{ [00], [010], [110] \} . \quad (11)$$

The numerical values for the quality of the destinations are as follows:

$$|RWS \in \Delta| = \frac{1}{2^{|[000]|}} + \frac{1}{2^{|[011]|}} + \frac{1}{2^{|[100]|}} + \frac{1}{2^{|[111]|}} , \quad (12)$$

$$= \frac{1}{2^3} + \frac{1}{2^3} + \frac{1}{2^3} + \frac{1}{2^3} = \frac{4}{8} = 0.5 , \quad (13)$$

$$|PBP \in \Delta| = \frac{1}{2^{|[00]|}} + \frac{1}{2^{|[010]|}} + \frac{1}{2^{|[110]|}} , \quad (14)$$

$$= \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^3} = \frac{4}{8} = 0.5 . \quad (15)$$

We now apply both client's preferences to calculate subjective qualities:

$$|RWS \in \Delta|_{Ann} = 0.25 \cdot 0.2 \cdot 0.5 + 0.25 \cdot 0.8 \cdot 0.5 \\ + 0.75 \cdot 0.7 \cdot 0.9 + 0.75 \cdot 0.3 \cdot 0.8 \quad (16)$$

$$= 0.025 + 0.1 + 0.4725 + 0.18 \quad (17)$$

$$= 0.7775, \quad (18)$$

$$|PBP \in \Delta|_{Ann} = 0.25 \cdot 0.2 + 0.25 \cdot 0.8 \cdot 0.5 + 0.75 \cdot 0.3 \cdot 0.2 \quad (19)$$

$$= 0.05 + 0.1 + 0.045 \quad (20)$$

$$= 0.195, \quad (21)$$

$$(22)$$

$$|RWS \in \Delta|_{Ben} = 0.85 \cdot 0.75 \cdot 0.05 + 0.85 \cdot 0.25 \cdot 0.2 \\ + 0.15 \cdot 0.1 \cdot 0.3 + 0.15 \cdot 0.9 \cdot 0.35 \quad (23)$$

$$= 0.031875 + 0.0425 + 0.0045 + 0.04725 \quad (24)$$

$$= 0.126125, \quad (25)$$

$$|PBP \in \Delta|_{Ben} = 0.85 \cdot 0.75 + 0.85 \cdot 0.25 \cdot 0.8 + 0.15 \cdot 0.9 \cdot 0.65 \quad (26)$$

$$= 0.6375 + 0.17 + 0.08775 \quad (27)$$

$$= 0.89525. \quad (28)$$

Thus, we obtained the value of 0.7775 as the measure of *Ann*'s interest in *RWS* and the value of 0.195 representing her interest in *PBP*, which is much lower. The value of 0.126125 confirms *Ben*'s aversion to spending time actively in comparison with the value of 0.89525, which reflects his strong interest in destinations allowing for a good rest and meals.

The results confirm the accuracy of our approach; they are consistent with common sense. As expected, for *Ann* the metaset model suggests *RWS*, where she is able to practise sports, and for *Ben* *PBP*, where he can have a rest and eat well. At this stage of development we cannot determine whether or not the proposed method is better than others. We will investigate this topic in the future and the results of the comparison will be publicized.

4 Generalization and Further Results

The definitions of metaset and related notions used in the paper are simplified versions of a much more general concept. Although first-order metasets are sufficient for the simple application discussed, for completeness we cite below the general definitions of *metaset* and *interpretation* (see [10] for a further discussion of metasets). The reader familiar with the method of forcing in set theory [3,7] will find some similarities here. They are rather superficial, since the prototype was designed for an entirely different purpose. Also, when applying metasets we are usually dealing with finite sets, which makes no sense in the case of forcing.

Definition 8. *A set which is either the empty set \emptyset or which has the form:*

$$\tau = \{ \langle \sigma, p \rangle : \sigma \text{ is a metaset, } p \in \mathbb{T} \}$$

is called a metaset.

Formally, this is a definition by induction on the well-founded relation \in (see [7, Ch. VII, §2] for a justification of this type of definitions). The general definition of interpretation for metasets is recursive as well.

Definition 9. *Let τ be a metaset and let $\mathcal{C} \subset \mathbb{T}$ be a branch. The set*

$$\tau_{\mathcal{C}} = \{ \sigma_{\mathcal{C}} : \langle \sigma, p \rangle \in \tau \wedge p \in \mathcal{C} \}$$

is called the interpretation of the metaset τ given by the branch \mathcal{C} .

For most applications, especially computer applications [9], first-order metasets - even finite first-order metasets - are sufficient. All the results presented here, together with the model itself, remain valid if we omit the assumption that the metasets involved are first-order. The definitions given above are used in the general development of the theory of metasets.

5 Conclusions

In this paper we explained a simple application of the new concept of sets with partial membership relation. We used metasets to model and solve the problem of selecting the best holiday destination for a client with specific preferences regarding how he spends his free time.

The metaset approach enables destinations and their properties to be rated using natural human-language terms. The core of the idea lies in constructing a treelike hierarchy of terms which describe the attributes of destinations and at the same time the requirements of clients. The hierarchy involves a relationship between attributes of the ‘generalization-specialization’ type.

Metasets are the perfect tool for evaluating imprecise expressions. The example we investigated here is that of a ‘perfect holiday destination’. Of course there is no one perfect place, just as there are no two persons having the same taste. The ideal place for one person to relax may give rise to resentment in another. This is a problem frequently encountered by the designers of mobile applications such as mobile tourist guides, which attempt to automatically determine which places to visit in a given region. The easiest way to determine the perfect location is to evaluate its popularity, i.e. how many people visit it or how many recommend it in polls or on Internet forums. In this way we overlook people with unusual preferences. Our approach eliminates this disadvantage, because the starting point of our algorithm is to identify user preferences and describe them in the form of a tree. Then the algorithm selects a location to suit those preferences. In addition, the test objects are set in a partial order, which will precisely take into account the needs of a specific person. Of course, the final

decision belongs to the user, but our algorithm can provide professional support. In our further work we will compare the approach presented in this paper with algorithms described in the research literature which are used for similar problems in logistics or tourism.

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