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MATHEMATICS
(COMPUTER SCIENCE)

Absolutely Computable Sets

by

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Summary. A function $f \in (R^k)^{\langle 0, +\infty)}$ is called a τ-computation iff there exists a continuous machine M of the delay τ such that f is a computation of M. It is proved that the set of all τ-computations in $\langle R^k)^{\langle 0, +\infty)}$ is equipotent to $(R^k)^{\langle 0, +\infty)}$. The paper contains an example of a function in $(R^k)^{\langle 0, +\infty)}$ which is not a τ-computation for any positive τ. A set of functions F is said to be τ-computable iff there exists a continuous machine M of the delay τ such that F is the set of computations of M. A set is said to be absolutely computable iff it is τ-computable for every positive τ. It is proved that the following sets are absolutely computable: the set of all entire functions of real variable, the set of all algebraic polynomials, the set of all trigonometric polynomials and the set of all solutions of a linear differential equation with constant coefficients.

1. Introduction

A continuous machine is a machine whose computations are real functions. Computations of continuous machines of the delay τ are called τ-computations. The basic properties of τ-computations were examined in [1]. Here we are concerned with some set-theoretic properties of the family $\{F_\tau^k\}$, where, for any natural k and positive τ, F_τ^k is the set of all τ-computations in $(R^k)^{\langle 0, +\infty)}$.

In [1] we have also examined some properties of τ-computable sets, being the sets of computations of continuous machines of the delay τ. The main part of this paper is devoted to absolutely computable sets. We say that a set of functions is absolutely computable iff it is τ-computable for every positive τ. Here we prove that some generally known sets of functions (for example, the set of all polynomials) are absolutely computable.

2. Preliminaries

In this section we shall recall some definitions and results from [1].

By Df and Rf we shall denote the domain and the range of function f, respectively. If $X \subset Df$, then by $f|_X$ we shall denote the restriction of f to X. The empty set will be denoted by $\emptyset$, the set of all real numbers by R and the set of all natural integer by N.

For any $k \in N$, by R^k we shall denote the k-dimensional Euclidean space, i.e.