

A THREE-VALUED LOGIC FOR SOFTWARE SPECIFICATION AND VALIDATION

Tertium tamen datur

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Abstract

Different calculi of partial or three-valued predicates have been used and studied by several authors in the context of software specification, development and validation. This paper offers a critical survey on the development of three-valued logics based on such calculi.

In the first part of the paper we review two three-valued predicate calculi, based on, respectively, McCarthy's and Kleene's propositional connectives and quantifiers, and point out that in a three-valued logic one should distinguish between two notions of validity: *strong validity* (always true) and *weak validity* (never false). We define in model-theoretic terms a number of consequence relations for three-valued logics. Each of them is determined by the choice of the underlying predicate calculus and of the weak or strong validity of axioms and of theorems. We discuss mutual relationships between consequence relations defined in such a way and study some of their basic properties.

The second part of the paper is devoted to the development of a formal deductive system of inference rules for a three-valued logic. We use the method of semantic tableaux (slightly modified to deal with three-valued formulas) to develop a Gentzen-style system of inference rules for deriving valid sequents, from which we then derive a sound and complete system of natural deduction rules. We have chosen to study the consequence relation determined by the predicate calculus with McCarthy's propositional connectives and Kleene's quantifiers and by the strong interpretation of both axioms and theorems. Although we find this choice appropriate for applications in the area of software specification, verification and development, we regard this logic merely as an example and use it to present some general techniques of developing a sequent calculus and a natural deduction system for a three-valued logic. We also discuss the extension of this logic by a non-monotone *is_true* predicate.

¹*Non-(tertium non datur)*